Physica Scripta. Vol. T118, 241–243, 2005

Emission Spectrum of an Electron in a Double Quantum Well Driven by Ultrashort Half-Cycle Pulses

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Received October 18, 2003; revised version received July 21, 2004; accepted August 13, 2004

PACS numbers: 42.65.Ky, 42.50.Hz, 78.67.De

Abstract

The emission properties of an electron in a double quantum well driven by ultrashort electromagnetic half-cycle pulses are studied. By numerically solving the corresponding time-dependent Schrödinger equation we show that the emission spectrum can be designed by appropriately choosing the parameters of the pulses. Low-frequency generation (LFG) is observed, and conditions for intense LFG and strong induced localization are found. The effects of absence of generalized parity of the Floquet modes on the emission spectrum are discussed. Simple relations for determining the values of the pulse parameters for controlling the localization process and the emission properties are found from an analytical approximation.

1. Introduction

A variety of novel phenomena such as coherent suppression of tunnelling [1–3], even harmonic generation [2, 3] and low frequency generation (LFG) [2, 3] among others, have recently been found in the interaction of electrons in symmetric double quantum wells with cw laser fields. From the practical viewpoint the control of the dynamics of a given quantum system can result in various applications, e.g. the laser-induced trapping of an electron in a quantum well [2, 3], the control of electron transfer reactions [4], the stabilization of a given configuration of a molecule [5, 6], and the creation of entangled states [7]. It can also be highly desirable for potential applications in designing electro-optical devices and is essential for the realization of quantum computation.

Although the emission and dynamical properties in double-well potentials [1–3] and in two-level systems [8, 9] has been theoretically explored in a considerable amount of works, all the studies have been limited to the case in which the driving field is a cw laser but no investigation has been reported on the possibility of controlling quantum coherence with a train of ultrashort half-cycle pulses (HCP) (for experimental works concerning generation and applications of HCPs, the reader can consult, for example, [10]). A highly asymmetric mono-cycle pulse is composed by a sharp tail with a high peak amplitude and a subsequent weak and smooth tail. As the peak amplitude of the sharp tail is several times larger than the smooth one, the dynamics of the system is mainly determined by the positive tail (for this reason the highly asymmetric pulse is called a half-cycle pulse). The generalized parity present in symmetric double wells driven by cw lasers is then absent in the case of HCPs as driving fields and each HCP delivers an impulsive momentum transfer (or *kick*) to the system [11].

In the present work we investigate the emission spectrum of an electron confined in a $Al_xGa_{1-x}As$ based double quantum well driven by a train of HCPs as well as its relation to the dynamics of the electron motion.

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2. General formulation

We consider a conduction electron confined in a typical $Al_xGa_{1-x}As$ based double quantum well growth in the z direction. Within the parabolic band and the effective mass approximations, the time-dependent Schrödinger equation describing the dynamics of the system under a train of HCPs can be written as

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t} = [H_0 + V_{conf} + V(z,t)]\Psi, \tag{1}$$

where H_0 represents the bare Hamiltonian, V_{conf} refers to the double-well confinement potential, and $V(z,t)$ corresponds to the interaction of the electron with the pulses. If the external field is periodic with period T the wave function of the system can be expressed as the superposition of the periodic Floquet modes $\Phi_\lambda(z,t)$ $[\Phi_\lambda(z,t) = \Phi_\lambda(z,t+T)]$ obeying the following equation [2]

$$\left[H_0 + V_{conf} + V(z,t) - \mathrm{i}\hbar\frac{\partial}{\partial t}\right]\Phi_\lambda = \varepsilon_\lambda\Phi_\lambda, \tag{2}$$

where the eigenvalues ε_λ represent the quasienergies.

A symmetric shape for the confinement potential similar to that in [3] was assumed. The electron effective mass $m^* = 0.067m_0$ was taken constant through the heterostructure.

The electron interaction with the train of strongly asymmetric pulses can be modelled by the potential

$$V(z,t) = z\sum_{k=0}^{N-1} F_k U(t - t_0 - kT), \tag{3}$$

where

$$U(t) = \begin{cases} \exp\left[-\dfrac{t^2}{2\sigma^2}\right]\cos\Omega t & \text{if } -\dfrac{\pi}{2\Omega} \le t < T - \dfrac{\pi}{2\Omega}. \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

In Eqs. (3) and (4) F_k denotes the peak field of the k-th pulse, t_0 corresponds to the time at which the first applied pulse is centered, T is the time between consecutive pulses, N is the number of applied pulses, and σ characterizes the width of the pulses. The parameter $\Omega = \frac{\pi}{3\sigma\sqrt{\ln 2}}$ in Eq. (4) guarantees a ratio 8:1 between the peak amplitudes of the positive and negative tails of the pulses. The duration d of the positive tail of each pulse is given by $d = 3\sigma\sqrt{\ln 2}$.

The time-dependent Schrödinger equation [Eq. (1)] can not be solved analytically, we therefore implemented a fast-Fourier-transform based numerical method as described in [12] for the time propagation of the initial wave function. After computation of the wave function $\Psi(z,t)$, quantities of interest as the probability $P_L(t) = \int_{-\infty}^{0} |\Psi(z,t)|^2\,\mathrm{d}z$ and the averaged probability $\langle P_L\rangle_\tau = \frac{1}{\tau}\int_0^\tau P_L(t)\,\mathrm{d}t$ of finding the electron in the left well can be easily computed. The emission properties are studied through the quantity $I(\omega) \sim |\int_{-\infty}^{\infty} \mu(t) \exp[-\mathrm{i}\omega t]\,\mathrm{d}t|$, where $\mu(t) = \langle \Psi(z,t)|z|\Psi(z,t)\rangle$ is the time-dependent dipole moment. All calculations were performed with $\sigma = 20\,\mathrm{fs}$ and $T = 100\,\mathrm{fs}$.

For a better understanding of the emission properties and their relation to the electron motion we developed, in addition to the numerical scheme, a simple analytical approach. We firstly note that for the system under study the two lowest-energy levels are well separated from the other energy states. Hence, for certain range of pulse parameters the system will behave, basically, as a two-level system. The two-level system is then studied within the sudden approximation (SA). The SA is valid if the duration of each pulse is much smaller than the characteristic time of the system (a condition that holds for the system here studied) and consists in substituting each actual pulse by an instantaneous *kick* that transfer a momentum Δp (Δp = area of the actual pulse) to the system. The electron-pulses interaction can then be approximated as $V(z,t) \approx z\sum_{k=0}^{N-1} \Delta p_k \delta(t - t_0 - kT)$ where $\delta(x)$ represents the Dirac delta function and $\Delta p_k = \int_{-\frac{\pi}{2\Omega}}^{T-\frac{\pi}{2\Omega}} F_k U(t)\,\mathrm{d}t$.

Within the two-level system approximation (TLSA) and the SA the quasienergies corresponding to the Floquet modes were found to be determined by $\varepsilon_\lambda = \epsilon_i + n\hbar\omega_0$ [$\omega_0 = 2\pi/T$; $\lambda = (i,n)$; $i = 1,2$; $n = 0, \pm 1, \pm 2, \ldots$], where

$$\epsilon_1 = -\frac{\hbar\omega_0}{2\pi} \arccos(\cos\alpha\cos\beta); \qquad \epsilon_2 = -\epsilon_1, \tag{5}$$

with $\alpha = \mu_{12}\Delta p/\hbar$ (μ_{12} is the dipole corresponding to the transition between the two lowest levels) and $\beta = \pi\omega_c/\omega_0$ (ω_c is the characteristic frequency of the bare system corresponding to the energy difference between the two lowest levels). The details on the TLSA and the SA will be given elsewhere.

3. Results

The time average of the probability P_L of finding the electron in the left well under the assumption that it was localized initially in that well (tunnelling initial conditions) as a function of the pulse strength is displayed in Fig. 1 (a). Solid and dashed lines correspond to the exact numerical result and the analytical approximation, respectively. A good agreement between both calculations can be appreciated, especially in the region of small pulse amplitudes (for very strong pulses the TLSA is no longer valid and the differences between the analytical model and the numerical results become larger). A remarkable fact is that, contrary to the case of a cw laser as a driving field, a train of HCPs can maintain the localization of the initially trapped particle in a wide range of pulse parameters.

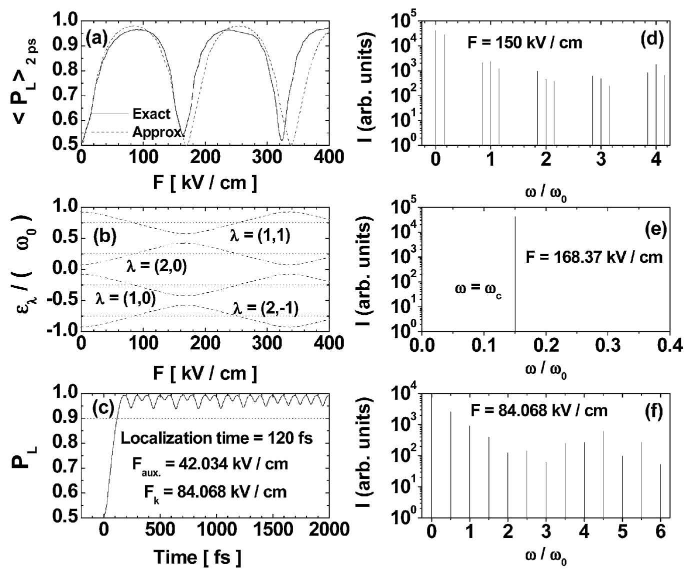

Fig. 1. (a) Time average of P_L as a function of the pulse strength for tunnelling initial conditions. (b) Dependence of the quasienergies on the pulse amplitude. (c) optimal localization process for the case of optical initial conditions. (d), (e), (f) Emission spectrum for different values of the pulse strength.

It is worth noting that when a train of HCPs is applied the Floquet modes of the system does not have a well defined generalized parity [note that for HCPs Eq. (2) is not invariant under the transformations ($z \to -z; t \to t + T/2$) as it is in the case of a cw laser]. Consequently, in the case of HCPs the existence of quasienergy crossings in the space of system parameters is no longer guaranteed. Indeed, for the system here studied the analytical approximation predicts a series of anti-crossings but no crossing (and, therefore, no accidental degeneracy) of quasienergies occurs. This situation is shown in Fig. 1 (b), where the dependence of the quasienergies (dashed lines) on the pulse amplitude is displayed. In the present case the achievement of localization in spite of the absence of accidental degeneracy of the quasienergies is due to the fact that for appropriate field parameters all the Floquet modes expanding the wave function of the system have the same phase at $t_0 + 2kT$ ($k = 0, 1, 2, \ldots$). Within the analytical model one can find that $P_L(t_0) \approx P_L(t_0 + 2kT)$ is guaranteed if $t_0 \approx T/2$ and

$$\varepsilon_\lambda = (n \pm \tfrac{1}{4})\hbar\omega_0 \tag{6}$$

(the details will be discussed elsewhere). If at $t = t_0$ the particle is still quasi-localized and the escaping time of the particle is much lunger than $2T$ then Eq. (6) will lead to a strong electron localization in the left well. The condition in Eq. (6) is represented by straight dotted lines in Fig. 1 (b). The comparison of Figs. 1 (a) and (b) confirms that the intersection of the quasienergies with the condition Eq. (6) determines the pulse amplitudes corresponding to optimal localization. On the other hand, as usually, delocalization occurs at the points of anti-crossing of the quasienergies [compare Figs. 1 (a) and (b)].

In practice, the common situation is that the initial state is the ground state of the field-free system in which the particle is completely delocalized (optical initial conditions). We then have studied the process of localizing the initially delocalized particle into the left well. Based on the analytical approximation we estimated the parameters of an auxiliary pulse that push the delocalized particle to the left well. After a time delay, when the particle localizes in the left well, a train of HCPs is used for maintaining the localization. The exact numerical results are shown in Fig. 1 (c), where a fast localization of the particle in the left well can be appreciated. This finding is also in sharp contrast to the case of cw lasers as driving fields, where it has not been possible to achieve such a strong and fast localization [3]. Thus, the use of HCPs for controlling the electron motion can be potentially useful for applications in designing electro-optical devices as efficient ultrafast switches.

The emission spectrum (vertical lines represent the emission peaks) obtained through exact numerical calculations for different values of the pulse strength is shown in Figs. 1 (d)–(f). For brevity we restrict the study of the emission properties to the case of tunnelling initial conditions. The emission spectrum is in general given by a static component at $\omega = 0$, integer harmonics ($\omega = n\omega_0$), and doublets $\omega = n\omega_0 \pm |\epsilon_2 - \epsilon_1|/\hbar$ around the integer harmonics. One can then design the emission spectrum by using

Eq. (5) for the estimation of the appropriate pulse parameters. The general case in which all the emission lines are present is shown in Fig. 1 (d), where the phenomena of LFG is quite apparent [note that the doublets corresponding to the n-th harmonic are close to the $(n \pm 1)$-th harmonics, and therefore the LFG line corresponds to the left doublet of the $n = 1$ harmonic]. Because of the absence of generalized parity, there is not accidental degeneracy of the quasienergies and hence there is a lower limit for the LFG determined by the lowest value of the difference $|\epsilon_2 - \epsilon_1|$ (note that this lowest value corresponds precisely to the characteristic frequency ω_c of the bare system), i.e., at the pulse parameters leading to optimal delocalization [see Figs. 1 (a) and (b)]. Under the situation of optimal delocalization only the line corresponding to LFG (that in this limit coincides with ω_c) survives, while the other lines collapse, i.e., the system behaves as *transparent* to the external field. This situation is shown in Fig. 1 (e). On the contrary, when Eq. (6) is fulfilled a crossing of the doublets corresponding to subsequent harmonics occurs at odd multiples of $\omega_0/2$. This situation corresponds to the process of optimal localization and the corresponding emission spectrum is displayed in Fig. 1 (f), where half-harmonic generation [i.e., at $\omega = n\omega_0/2$ $(n = 0, 1, 2, 3, \ldots)$] can be clearly appreciated.

In summary, we have shown that a conveniently designed train of HCPs can be used for an efficient control on the subpicosecond scale of the electron motion (note that such a process lasts several picoseconds when cw lasers are used as driving fields [3]) in symmetric double quantum wells. The emission properties of the system as well as their relation to the electron motion were studied and useful relations for estimating the pulse parameters that control the emission spectrum were found by means of an approximated analytical model.

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