Ultrafast control of electronic motion in semiconductor nano and mesoscopic structures

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ABSTRACT

We study the charge dynamics in a double quantum well and in ballistic mesoscopic rings driven by half-cycle pulses. It is shown that such pulses can be utilized to localize, within femtoseconds, and control, for picoseconds, the electronic motion in a $Ga_{1-x}Al_xAs$ based double quantum well. To identify the pulse parameters that appropriate for an efficient control process we developed a simplified analytical model and corroborated the results by performing full numerical calculations. We also show that when a thin ballistic mesoscopic ring is subjected to a linearly polarized HCP a post-pulse (and therefore field-free) polarization is induced in the ring. The non-equilibrium post-pulse polarization oscillates in the ring as long as the coherence is preserved and decays on a time scale determined by the relaxation time.

Keywords: Ultrafast control, Half-cycle pulses, Mesoscopic systems

1. INTRODUCTION

The time-dependent electric field in a sub-picosecond unipolar electromagnetic pulse resembles one-half of an optical field cycle in an electromagnetic wave and therefore such pulses are often referred to as "half-cycle" pulses (HCPs) (Fig.1 b).[1] In principle, for a freely propagating electromagnetic wave the time integral over the electric field $\mathbf{F}(t)$ vanishes. An HCP makes non exception, however the half cycle with the opposite polarity is strongly attenuated and stretched in time (cf.Fig.1 b). Therefore, in reality HCPs are mono-cycle pulses consisting of a very short, half cycle with a duration t_d followed by a much slower half-cycle of an opposite polarity and a much smaller amplitude (the tail of the pulse). Typical pulse amplitude asymmetry is 13:1.[1]

The interaction of charge carriers with HCPs differs fundamentally from that with continuous wave (CW) lasers: if the characteristic time scale of carriers motion (such as the round-trip time of a confined electron) is longer that the HCP duration, the interaction with HCP can be viewed as an impulsive "kick" received by the electron (as outlined in the appendix).[2] For times longer than the HCP duration t_d but shorter than the duration of the complete pulse, the amount of the kick (i.e. the charge carrier momentum change) is given by $\Delta p = -\int_0^{t_d} \mathbf{F}(t)dt$. This physical picture is reflected in the quantum dynamics of the particles: subjecting a particle to an HCP leads in effect to a linear transformation of the momentum space wave function $\tilde{\Psi}(\mathbf{p})$ in the direction of the kick, i.e. $\tilde{\Psi}(\mathbf{p}) \rightarrow \tilde{\Psi}(\mathbf{p}+\Delta p)$. In the configuration space, the application of an HCP phase-shifts the electron wave function according to $\Psi(\mathbf{r}) \rightarrow \Psi(\mathbf{r})e^{-i\Delta p \cdot \mathbf{r}}$. From this scenario of the electron-HCP interaction one may expect that the position and the momentum of a given electronic distribution can be controlled and manipulated (to a certain degree quantified below) by applying a sequence of kicks with appropriate relative strengths, delays, and directions. Such HCP trains are feasible nowadays.[3]

This paper presents two cases demonstrating how the charge carriers behave when exposed to HCPs:
1) we investigate the electron dynamics in a double quantum well (DQW) driven by a sequence of HCPs. We find that an appropriately designed train of HCPs renders possible a swift control (in femtoseconds) of the electron motion in symmetric double-well structures. This finding is in contrast to the case when CW lasers are used as driving fields,[4] in which case such a strong localization was not achieved and the time needed to reach the electron localization is on the order of few picoseconds.[4]
2) As a second example we study the dynamics of a thin ballistic mesoscopic ring (MR) subjected to a linearly polarized HCP. Over the past few decades considerable research efforts have been devoted to the study of mesoscopic systems which constitute a paradigm for the intriguing manifestation of quantum mechanics on a macroscopic (micrometer) scale. At low temperatures the phase coherence length of the charge carriers in such systems increases significantly and may well exceed or be on the order of the system size. Quantum interferences

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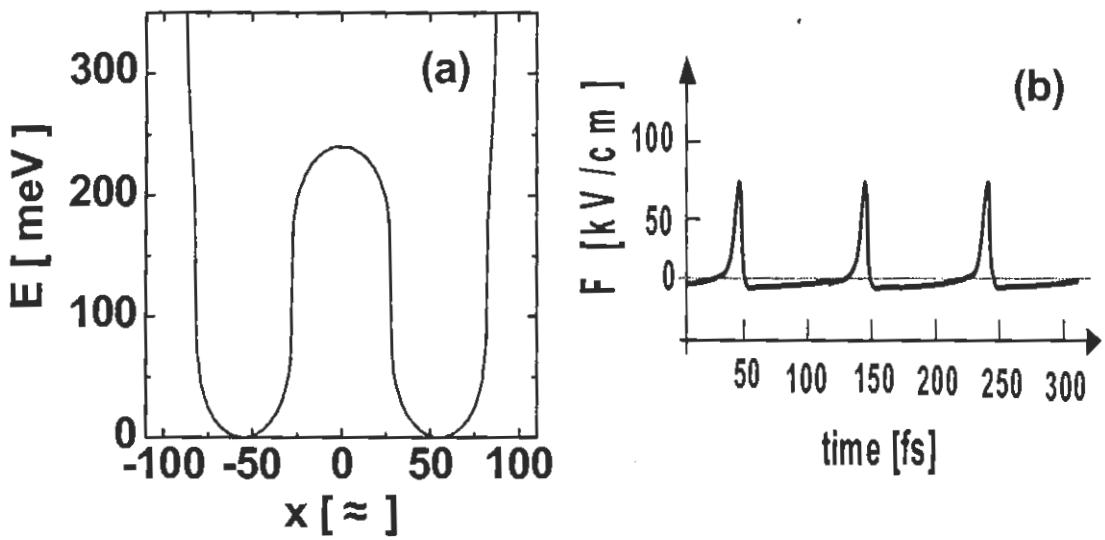

Figure 1. (a) Electron confining potential: the central barrier hight is $\sim$ 240 meV. The wells and barrier widths are $\sim$ 50 Å and $\sim$ 60 Å, respectively. (b) The electric field amplitude vs. time for a typical (realizable) sequence of half cycle pulses.

dominates then the behaviour of the system giving rise to phenomena such as the persistent curents[5–11] and the Aharonov-Bohm conductance oscillations[12] in MRs. A further aspect that was addressed by previous studies is the dynamics of charge carriers in MR under the action of time-dependent electric fields while the MR is being threaded by a static magnetic field.[13–16] Some investigations were devoted to the dynamical properties of MRs driven by CW lasers.[17–19] Here we focus on the study of the charge polarization of a MR induced by an HCP. That a time-averaged charge polarization can be induced at all is due to the strong (time) asymmetry of the HCP, as outlined above. A CW field does not induce in the ring a time-averaged (over the period of the laser) polarization. The non-equilibrium charge polarization generated by HCP oscillates in time (as detailed below) and lasts for times much longer than the pulse duration, offering thus the possibility to study non-equilibrium states in absence of external fields.

2. DYNAMICAL LOCALIZATION IN DOUBLE QUANTUM WELLS

We consider a conduction electron confined in a typical $Al_xGa_{1-x}As$ based DQW. Within the parabolic band and the effective mass approximations (the effective mass $m^* = 0.067m_0$ is assumed constant throughout the heterostructure), the time-dependent Schrödinger equation describing the dynamics of the system driven by a sequence of HCPs is given by

$$i\hbar\frac{\partial\Psi}{\partial t} = H\Psi \quad ; \quad H = H_0 + V_{conf} + V(x,t) \tag{1}$$

where H_0 is the bare Hamiltonian, V_{conf} refers to the confinement potential [cf. Fig. 1 (a)] and $V(x,t)$ stands for the coupling of the electron to the pulses. As shown below a typical localization time is $\sim$ 132 fs which is well below the typical time scale (several picoseconds) for the elastic scattering and electron-phonon interaction in high purity Ga(Al)As-GaAs heterostructures with typical electron concentrations.[20] Therefore, these effects are subsidiary for the localization process.

The electron interaction with a sequence of N_p HCPs is described by the time-dependent potential

$$V(x,t) = x\sum_{k=1}^{N_p} F_k a(t-t_k) \quad , \tag{2}$$

where F_k denotes the peak field of the kth pulse. All the pulses are considered to be described by the same envelope function $a(t)$ and centered at times t_k ($k = 1, ..., N_p$). For the envelope shape we use a Gaussian function, i.e., $a(t) = \exp[-t^2/(2\sigma^2)]$ with $\sigma = 10$ fs. This means that the relevant (positive polarity) part of the HCPs envelop [Fig. 1 (b)] is modelled by the Gaussians, performing the calculations with the actual pulses shown in Fig. 1 (b) leads to essentially the same results presented below.

The time-dependent Schrödinger equation (1) with the driving potential (2) is solved numerically using a fast-Fourier-transform based numerical method[21] for the time propagation of the initial wave function. We assume that the system is initially in the ground state of the field-free symmetric DQW. Then, before applying the pulses, the particle is completely delocalized, with the same probability of being in the left or in the right well. In order to trace the electron motion upon the application of the HCPs, we investigate the time evolution of the dynamic probability

$$P_L(t) = \int_{-\infty}^{0} \Psi^*(x,t)\Psi(x,t)dx \tag{3}$$

of finding the electron in the left well.

We are particularly interested in the femtosecond control of the electron wave-packet localization in the double well heterostructure by means of a train of HCPs. As a general strategy for controlling the electron motion we propose the following scheme. One applies at first (at $t = t_1$) an auxiliary HCP with a strength F_{aux} capable of promoting the system from the ground state to a time-dependent coherent state for which the electron localizes in one of the well, say in the left well. The localization is achieved at the time $t = t_1 + t_{loc}$; the time lag t_{loc} we call hereafter the localization time. Once the electron has been localized in the left well one applies a periodic train of HCPs for suppressing the electron tunnelling to the right well. In this way, the electron localization will be sustained in time until the train of pulses is turned off. The main task is then to find the appropriate pulse parameters that lead to a sustainable localization of the electron in one of the wells. To obtain an estimate on such field parameters we developed, in addition to the exact numerical calculation, a simplified analytical approach. The analytical approximation is based on the following observations. For the system under study the two lowest-energy levels are well separated from the other energy states. Hence, for a certain range of pulse parameters the system will behave, basically, as a two-level system. Further simplification is brought about by the fact that for ultrashort HCPs the duration of each pulse is much smaller than the typical characteristic time $\tau_c = 2\pi/\omega_c$ (ω_c is the frequency corresponding to the energy difference between the ground and the first excited states of the field-free DQW) of the undriven system (in the DQW studied here, we have for example $\tau_c \approx 665$ fs in absence of the pulses, while the duration of the employed pulses is about 50 fs). As the width of the pulses is very small compared to the characteristic time of the undriven system, one can apply the impulsive approximation (IA).[22] For details on the IA see Appendix A.

Within the two-level approximation the wave function of the system can be expressed as

$$\Psi(x,t) = \sum_{n=1}^{2} C_n(t)\Psi_n^{(0)}(x) \ , \tag{4}$$

where $\Psi_n^{(0)}(x)$ ($n = 1, 2$) represent the two lowest levels of the field-free system. The two-dimensional spinor $\mathbf{C}(t) = (C_1(t), C_2(t))^T$ obeys the time-dependent Schrödinger equation

$$i\hbar\frac{\partial \mathbf{C}(t)}{\partial t} = \left[-(\hbar\omega_c/2)\sigma_z + \mu_{12}\sum_{k=1}^{N_p} F_k a(t-t_k)\sigma_x\right]\mathbf{C}(t) \ , \tag{5}$$

where σ_x and σ_z are Pauli matrices and $\mu_{12} = \langle\Psi_1^{(0)}(x)|x|\Psi_2^{(0)}(x)\rangle$.

From Eqs. (3) and (4), and taking into account the symmetry of the system one finds that within the two-level system approximation the time evolution of the probability of finding the electron in the left well is given by

$$P_L(t) = \frac{1}{2} + \mathrm{Re}[C_1^*(t)C_2(t)] \ . \tag{6}$$

After application at $t = t_1$ of the auxiliary pulse the two-dimensional spinor $\mathbf{C}(t) = (C_1(t), C_2(t))^T$ evolves as (see Appendix A)

$$\mathbf{C}(t) = U_0(t, t_1)U(t, t_1, 0)U_0(t_1, 0)\mathbf{C}(0) \ , \tag{7}$$

where

$$U_0(t, t') = \begin{pmatrix} e^{\frac{i}{2}\omega_c(t-t')} & 0 \\ 0 & e^{-\frac{i}{2}\omega_c(t-t')} \end{pmatrix} \ , \tag{8}$$

$$U(t, t_1, 0) = e^{\frac{i}{\hbar}p_1\sigma_x} = \begin{pmatrix} \cos\left(\frac{\mu_{12}p_1}{\hbar}\right) & i\sin\left(\frac{\mu_{12}p_1}{\hbar}\right) \\ i\sin\left(\frac{\mu_{12}p_1}{\hbar}\right) & \cos\left(\frac{\mu_{12}p_1}{\hbar}\right) \end{pmatrix} \ , \tag{9}$$

and $p_1 = p_{aux} = F_{aux}\int a(t - t_1)dt$ is the momentum transferred to the system by the auxiliary pulse. For the case of a Gaussian-like pulse the transfer of momentum due to the auxiliary pulse is given by $p_{aux} = F_{aux}\sigma\sqrt{2\pi}$.

Assuming that the system is initially in the ground state of the undriven DQW [i.e., $\mathbf{C}(t) = (1, 0)^T$] one obtains from Eqs. (6) - (9) that for an auxiliary pulse with peak amplitude such that $\mu_{12}p_1/\hbar = \pi/4$, the probability $P_L(t_1 + \tau_c/4) = 1$, i.e., with a time lag $(\tau_c/4)$ upon the application of the auxiliary pulse, the electron is localized perfectly in the left well. The value $\tau_c/4$ is then the rough estimate for the localization time. The appropriate peak field for a Gaussian-like auxiliary pulse to localize the initially delocalized electron can be estimated as

$$F_{aux} = \frac{\hbar\sqrt{2\pi}}{8\mu_{12}\sigma} \ . \tag{10}$$

In absence of external fields the localized electron tunnels to the other well (more precisely, the peak value of the electron probability density oscillates from one well to the other with a period τ_c). So the localization can not be sustained for times intervals longer than $\tau_c/4$. In order to suppress coherently the tunnelling we apply a periodic train of HCPs at $t_2 = t_1 + \tau_c/4 + \gamma$. The value of $\gamma \ll \tau_c/4$ can be chosen arbitrarily subject to the restriction $\gamma \ll \tau_c/4$ (this restriction prevents the tunnelling of the electron before the application of the train of pulses).

The train of HCPs is assumed to be periodic with a period T and to be built out of individual pulses with peak amplitudes $F_k = F$, $k = 2, 3, ..., N_p$. The evolution of the system after the application of the train of HCPs is given, at stroboscopic times $[t = t_2 + (k - 2)T;\ k = 2, ..., N_p]$ by the relation

$$\mathbf{C}(t_2 + (k-2)T) = [U(t_2 + T, t_2)]^{k-2}\mathbf{C}(t_2) \quad ; \quad k = 2, ..., N_p \ , \tag{11}$$

where the operator $U(t_2 + T, t_2)$ is given by

$$U(t_2 + T, t_2) = U_0(t_2 + T, t_2)U(t_2 + T, t_2, t_2) = \begin{pmatrix} e^{\frac{i}{2}\omega_c T}\cos\left(\frac{\mu_{12}p}{\hbar}\right) & ie^{\frac{i}{2}\omega_c T}\sin\left(\frac{\mu_{12}p}{\hbar}\right) \\ ie^{-\frac{i}{2}\omega_c T}\sin\left(\frac{\mu_{12}p}{\hbar}\right) & e^{-\frac{i}{2}\omega_c T}\cos\left(\frac{\mu_{12}p}{\hbar}\right) \end{pmatrix} \ , \tag{12}$$

and $p = F\int a(t - t_2)dt$. Note that in Eq. (11) $\mathbf{C}(t_2)$ refers to the electron being localized in the left well. For sustaining this localization in time we then propose to induce a periodic cyclic evolution to the system, i.e., to drive the system through a time dependence such that

$$\mathbf{C}(t_2 + l\mathcal{T}) = e^{i\phi_l}\mathbf{C}(t_2) \quad ; \quad l = 0, 1, 2, ... \ , \tag{13}$$

where ϕ_l is a real number (the phase acquired by the wave function within the lth evolution cycle) and $\mathcal{T}$ represents the duration of one evolution cycle (note that $\mathcal{T}$ does not necessarily coincide with T). It is clear from Eqs. (6) and (13) that if the system undergoes a periodic cyclic evolution then $P_L(t)$ becomes periodic with period $\mathcal{T}$, i.e.,

$$P_L(t_2 + l\mathcal{T}) = P_L(t_2) \quad ; \quad l = 0, 1, 2, ... \ . \tag{14}$$

Thus, if the particle was localized in the left well at $t = t_2$ (i.e., if $P_L(t_2) \approx 1$) then it will also localize in that well at any time $t = t_2 + l\mathcal{T}$. By imposing the condition in Eq. (13) on Eq. (11) one obtains that for a periodic train of HCPs with peak amplitudes such that $\mu_{12}p_k/\hbar = (2n + 1)\pi/2;\ n \in \mathcal{Z}$, the system follows a cyclic evolution

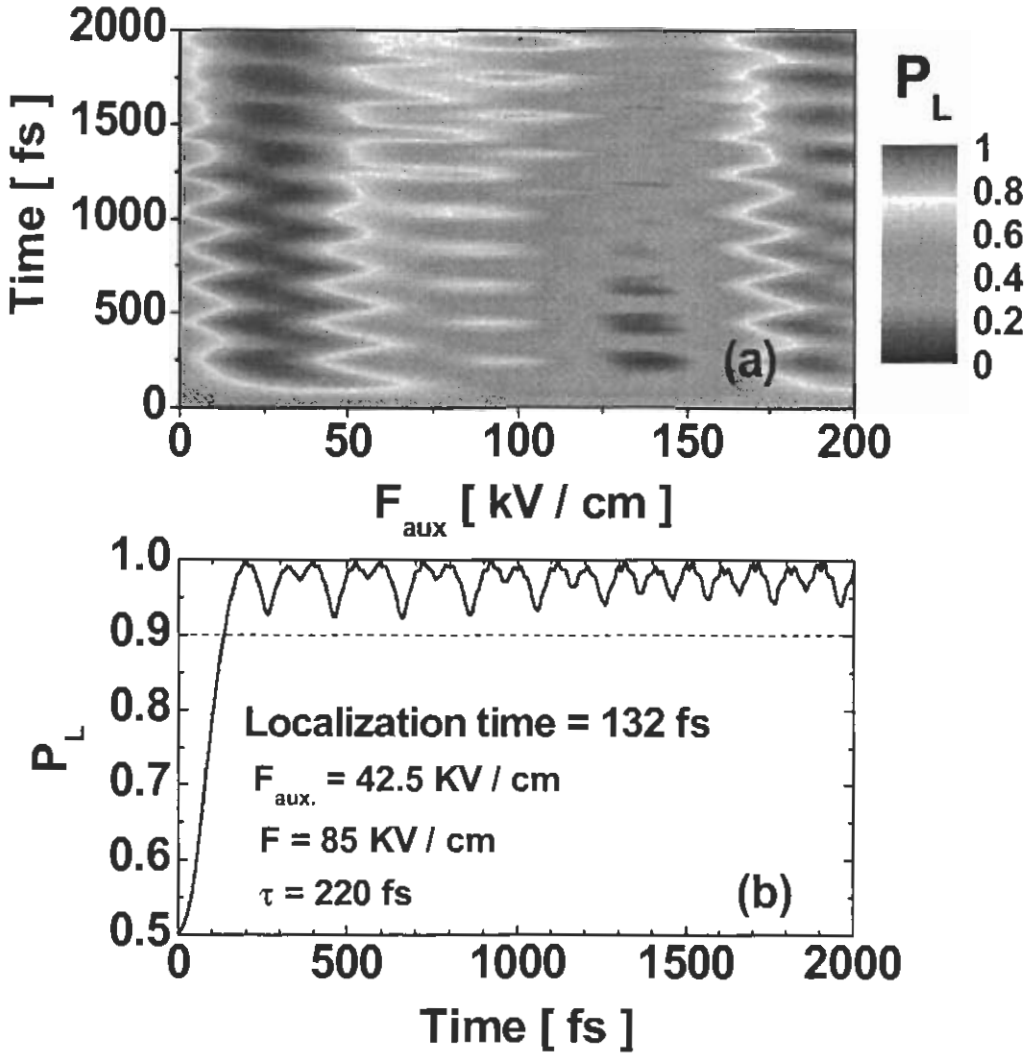

Figure 2. a) P_L vs. time and peak amplitude F_{aux} of the auxiliary pulse. Following F_{aux} we apply, after a time delay $\tau = t_2 - t_1 = 220$ fs, a quasiperiodic train of HCPs with peak amplitudes $F_k = 85$ kV/cm (see text for details). (b) a cut in (a) at $F_{aux} = 42.5$ kV/cm. The values of the peak amplitudes were estimated by using Eqs. (10) and (15). The period of the train of HCPs is set to be $T = 100$ fs.

with $\mathcal{T} = T$ or $\mathcal{T} = 2T$ in dependence of whether $\gamma = T/2$ or $\gamma \neq T/2$. The appropriate peak amplitudes F_k for the train of Gaussian-like HCPs to suppress the electron tunnelling can then be estimated as

$$F_k = F = \frac{(2n+1)\hbar\sqrt{2\pi}}{4\mu_{12}\sigma} \;\; ; \;\; k = 2, 3, ..., N \;\; ; \;\; n \in \mathcal{Z} \;\; . \tag{15}$$

We remark that the condition above has to be complemented with the requirement $T \ll \tau_c/4$, in order to avoid the tunnelling of the trapped particle between consecutive pulses.

We summarize the ideas of the scheme presented here. An auxiliary pulse with peak amplitude F_{aux} given by Eq. (10) is applied at $t = t_1$. The auxiliary pulse pushes the electron into the left well. Once the particle has been localized in the left well, a subsequent train of HCPs with period $T \ll \tau_c/4$ (for the numerical calculations we assumed $T = 100$ fs) and field amplitudes determined by Eq. (15) is applied at $t = t_2$. The train of HCPs then keeps the particle localized in that well by "kicking" it back at the time when it starts tunnelling to the

second well. Following this strategy we performed full numerical calculations. The results (see Fig. 2), evidence that strong localization of the initially delocalized electron can be achieved in times of the order of a hundred femtoseconds. This finding is in sharp contrast to the case when CW lasers are used as driving fields,[4] where it has not been possible to achieve such a strong localization and, in addition, the time needed to achieve electron localization was found to be on the order of few picoseconds.[4] We also note that although we have focused in the localization in the left well, the electron localization in the right well can be achieved by changing the polarity of the HCPs. Furthermore, as clear from Fig. 2(a) the localization is robust to considerable changes in the field strength which makes the present control scheme a good candidate for applications, such as the design of electro-optical devices and ultrafast switches.

3. CHARGE POLARIZATION OF DRIVEN MESOSCOPIC RINGS

We consider a one-dimensional, non-interacting MR of radius ρ_0 at zero temperature exposed (at $t = t_1 = 0$) to a single HCP which is linearly polarized along the x direction. The ring is considered to be isolated and to contain N spin-$\frac{1}{2}$ particles. The applied pulse has a duration τ_d much shorter than the ballistic time τ_F which is the time interval a particle at the Fermi level (E_F) needs for completing one turn around the ring. This condition $\tau_d \ll \tau_F$ is currently feasible experimentally: for a typical ballistic ring τ_F is several tens of picoseconds[23,24] and HCPs with $\tau_d = 1$ ps are readily available.[1] The problem can then be treated within the impulsive approximation (see Appendix A). Within this approximation, the single-particle wave function just before ($t = 0^-$) and right after ($t = 0^+$) the pulse are related through the following matching condition

$$\Psi(\theta, t = 0^+) = \Psi(\theta, t = 0^-) e^{i\alpha \cos\theta} \quad . \tag{16}$$

where e denotes the electron charge, $\alpha = e\rho_0 p/\hbar$ and p is the area of the pulse (i.e. the time-integral over the electric filed amplitude of the HCP). The polar angle θ defines the angular position of the charge carrier with respect to the HCP polarization axis.

The time-dependent wave function $\Psi_{m_0}(\theta, t)$ describing the evolution of a particle initially residing in the m_0th orbital state can be expanded in terms of the ring stationary eigenstates $\Psi_m^{(0)}(\theta) = e^{im\theta}/\sqrt{2\pi}$ as

$$\Psi_{m_0}(\theta, t) = \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} C_m(m_0, t) e^{im\theta} e^{-i\frac{E_m t}{\hbar}} \quad . \tag{17}$$

Here we denoted the orbital energies of the unperturbed states by E_m, where

$$E_m = \frac{\hbar^2 m^2}{2m^* \rho_0^2} \quad , \quad m = 0, \pm 1, \pm 2, \cdots \quad . \tag{18}$$

Taking into account the matching condition stated in Eq. (16) and after applying the expansion theorem one finds that the expansion coefficients are given by

$$C_m(m_0, t) = \begin{cases} \delta_{m,m_0} & \text{for } t \le 0 \\ i^{m_0 - m} J_{m-m_0}(\alpha) & \text{for } t > 0 \end{cases} \quad , \tag{19}$$

with $J_l(x)$ representing the Bessel functions and $\delta_{m,n}$ is the Kronecker symbol.

Upon applying the HCP the energy spectrum of the particles is rearranged. Specifically, the energy corresponding to a particle initially in the m_0th state evolves as

$$\begin{aligned} E_{m_0}(t) &= \langle \Psi_{m_0}(\theta, t) | H | \Psi_{m_0}(\theta, t) \rangle, \\ &= i\hbar \left\langle \Psi_{m_0}(\theta, t) \left| \frac{\partial}{\partial t} \right| \Psi_{m_0}(\theta, t) \right\rangle \quad . \end{aligned} \tag{20}$$

From Eqs. (17) - (20) one finds that the energy corresponding to a particle initially in the m_0th state is given by

$$E_{m_0}(t) = \begin{cases} \frac{\hbar^2 m_0^2}{2m^* \rho_0^2} & \text{for } t \le 0 \\ \frac{\hbar^2}{2m^* \rho_0^2} \left(m_0^2 + \frac{\alpha^2}{2} \right) & \text{for } t > 0 \end{cases} \quad . \tag{21}$$

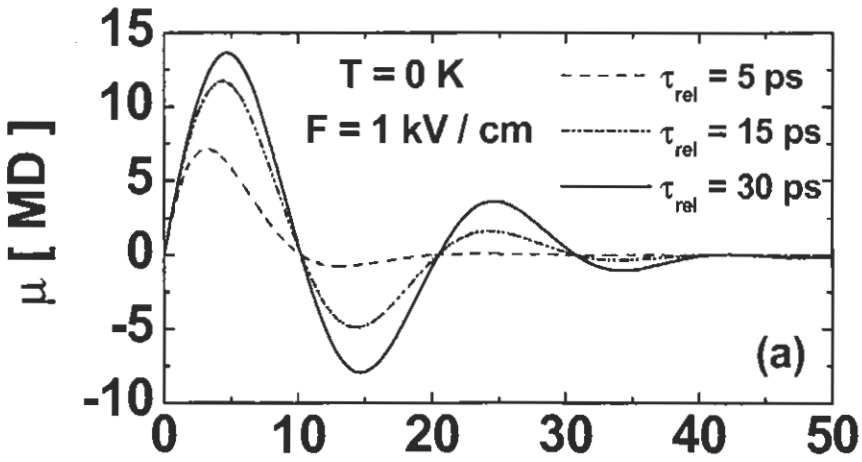

Figure 3. Time dependence of the dipole moment μ for different values of the relaxation time τ_{rel}. The radius of the ring and the number of particles were set to $\rho_0 = 1.5$ μm and $N = 1600$, respectively.

Recalling that $\alpha = q\rho_0 p/\hbar$ we can write for the particle energy upon the pulse

$$E_{m_0}(t > 0) = E_{m_0}(t < 0) + \frac{q^2}{2}\frac{p^2}{2m^*}. \tag{22}$$

Thus, applying an HCP to the ring shifts the unperturbed energy spectrum by an amount that scales quadratically with the strength of the pulse and does not depend on the size of the ring. The initial energy degeneracy (of the clock- and anticlock-wise circulating angular states) is preserved after the pulse is applied. Furthermore, since $E_{m_0}(t > 0)$ grows quadratically with m_0 Eq. (22) dictates that the energy of a particle at the Fermi level (for which $m_0 \approx N/4$) is affected only marginally by the pulse if $(N/4)^2 \gg \alpha^2/2$. For small p and for rings containing a large number of particles this condition may well be satisfied, e.g., as for the explicit numerical illustrations discussed below.

To inspect the structure of the coherent states created by the pulse we analyze Eqs. (17) and (19). Under the condition $m' = |m - m_0| \gg \alpha$ we have from Eqs. (19) and the asymptotic behavior of the Bessel functions that

$$|C_m(m_0, t > 0)| \approx \frac{1}{\sqrt{2\pi m'|}}\left(\frac{e\alpha}{2m'}\right)^{m'} \quad ; \quad m' \gg \alpha\,. \tag{23}$$

In the weak field regime, i.e. for small p, α is small. In such a situation the condition $m' = |m - m_0| \gg \alpha$ is easily reached and $|C_m(m_0, t > 0)|^2$ rapidly decays when increasing the value $|m - m_0|$, i.e., only few states (labelled by m) around m_0 contribute to the coherent population created by the pulse.

It is not difficult to prove from Eqs. (17) - (19) that $\Psi_{m_0}(\theta, t) = \Psi_{-m_0}(-\theta, t)$, i.e., the clockwise-counterclockwise symmetry of the charge carrier is preserved after the application of the pulse, and therefore, currents carried by particles initially in the m_0 and $-m_0$ states compensate each other. This fact together with the degeneracy of the states [see Eq. (21)] confirms the intuitive expectation that no total (time-averaged) current is induced in the ring.

The dipole moment μ_{m_0} along the x axis corresponding to a particle initially in the m_0th stationary state is is given by

$$\mu_{m_0}(t) = q\rho_0\langle\cos\theta\rangle_{m_0}(t)\ , \tag{24}$$

where

$$\langle\cos\theta\rangle_{m_0}(t) = \int_0^{2\pi} |\Psi_{m_0}(\theta, t)|^2 \cos\theta d\theta\ . \tag{25}$$

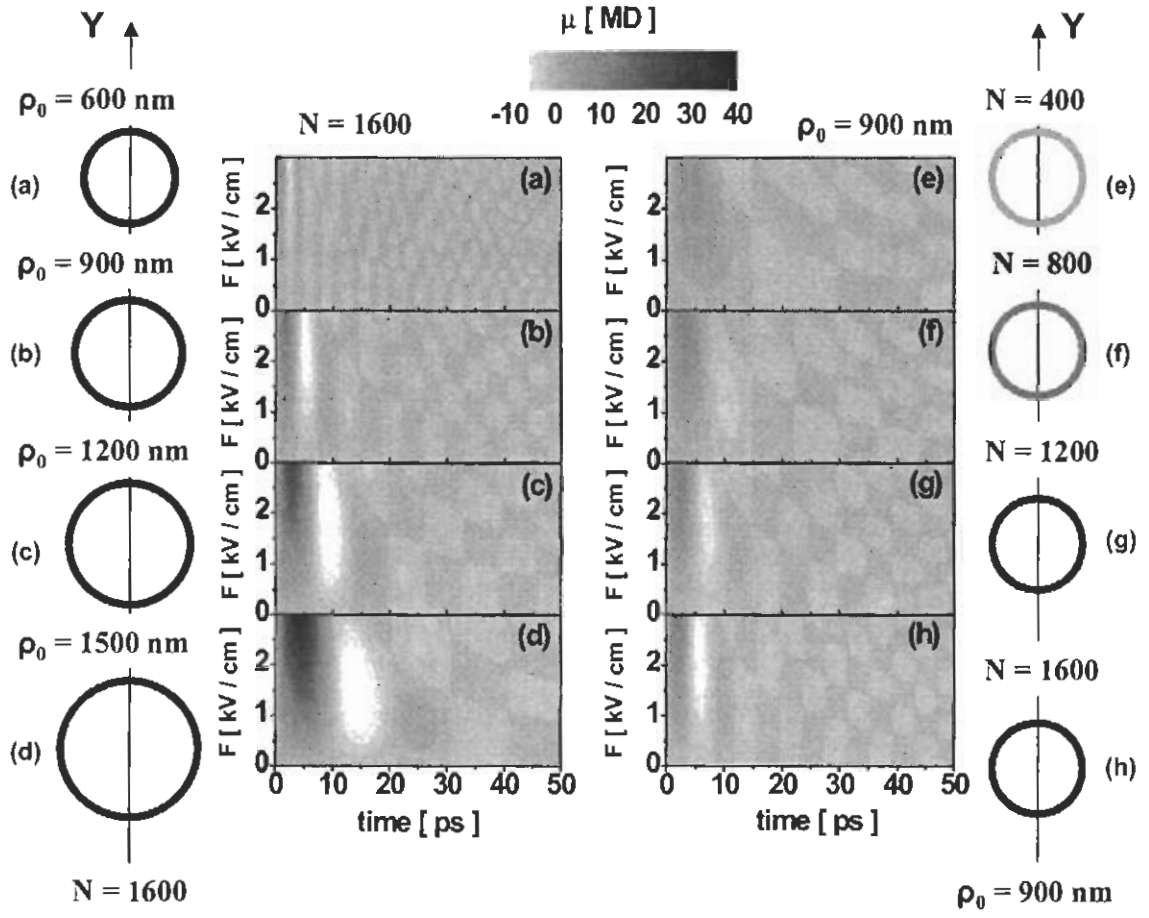

Figure 4. Time dependence of the total dipole $\mu(t)$ as a function of the pulse field amplitude F for an array of eight non-interacting rings with varying radii [(a) - (d)] and number of particles [(e) - (h)] at zero temperature.

From Eqs. (17) - (19), and (25) we deduce that $\langle \cos\theta \rangle_{m_0}(t) = \langle \cos\theta \rangle_{-m_0}(t)$. Therefore, the contributions of particles initially in the m_0 and $-m_0$ states to the polarization interfere constructively and a non-vanishing total polarization is generated. The total HCP-induced dipole moment along the x axis is given by

$$\mu(t) = \sum_{m_0,\sigma} f(m_0,t)\mu_{m_0}(t) \quad , \tag{26}$$

where σ refers to the spin of the particle, f represents the non-equilibrium distribution function, and $\mu_{m_0}(t)$ is given by Eq. (24). In the weak-field limit considered here, the non-equilibrium distribution function can be calculated within the relaxation time approximation.[25] The corresponding Boltzmann equation for determining f is given by

$$\frac{\partial f(m_0,t)}{\partial t} = \frac{f(m_0,t) - n_F(m_0)}{\tau_{rel}} \quad , \tag{27}$$

where τ_{rel} represents the (averaged) relaxation time and $n_F(m_0)$ denotes the Fermi-Dirac distribution function corresponding to the equilibrium state.

We performed explicit calculations for a ballistic GaAs-AlGaAs ring of the type used in the experiment reported in Ref. 24. The electron effective mass was set $m^* = 0.067 m_e$. Sine-square shaped HCPs with a time duration of 1 ps were considered. All the calculations correspond to the zero temperature regime.

The time dependence of the total dipole moment μ for different values of τ_{rel} are shown in Fig. 3. We recall that the duration of the pulse is 1 ps. Taking this into consideration we conclude that the build-up and decay of the polarization, as illustrated in Fig. 3, occur in a field-free manner, e.g., the polarization is generated within 10 ps after the application of the pulse. This peculiarity can be exploited as a unique opportunity for investigating relaxation processes in absence of external perturbations. It is also noteworthy that the magnitude of μ is substantial [μ in Fig. 3 is depicted in units of 10^6 D]. The maximum absolute value of the induced electric dipole moment decreases when shortening the relaxation time. Nevertheless, the post-pulse polarization is still appreciable within a typical range of values of τ_{rel} in ballistic semiconductor MRs[5] as shown in Fig. 3.

A further potential application of the proposed scheme is to apply a pulse (or train of pulses) to an artificially planar array of isolated rings. Such rings arrays (however interconnected) have been realized experimentally, as reported in Ref. 26. Upon an appropriate design of the corresponding planar array (e.g., by varying the radius or the number of particles in certain rings in the array) one can create a desired map of a charge polarization of the quasi two-dimensional structured material. This situation is illustrated in Fig. 4 for an array of eight isolated rings.

4. CONCLUSIONS

In summary, we showed that the electron motion in a symmetric DQW can be controlled on a femtosecond scale by subjecting the system to an appropriately designed sequence of HCPs. Simple analytical expressions for estimating the pulse parameters leading to the dynamical localization of the electron were obtained by using a two-level system approximation. The analytical results were corroborated by implementing full numerical calculations. We expect this efficient control scheme to be potentially interesting to various applications, in particular in designing ultrafast electro-optical devices. We also showed that the application of a linearly polarized HCP to a ballistic MR induces a post-pulse (and therefore field-free) polarization. The time-dependent post-pulse polarization could be useful for measuring relaxation times since it oscillates in the ring and decays in a time of the order of τ_{rel}. Furthermore, planar arrays of isolated MRs driven by HCPs could be useful for designing artificially structure materials with desired polarization properties.

APPENDIX A. THE PROPAGATOR WITHIN THE IMPULSIVE APPROXIMATION

We consider a system described by the Hamiltonian $H^{(0)}$. The system is subject to an electromagnetic pulse centered at $t = t_1$. The subsequent time evolution of the system is determined by the time-dependent Hamiltonian $H = H^{(0)} + V(t)$, where $V(t)$ represents the coupling of the external field to the system. The time evolution operator $U(t,0)$ satisfies the equation of motion

$$i\hbar\frac{\partial U(t,0)}{\partial t} = [H^{(0)} + V(t)]U(t,0) \ , \tag{28}$$

and satisfies the relation

$$U(t,t_0) = U_0(t,t_1)U(t,t_1,0)U_0(t_1,0) \ , \tag{29}$$

where the propagator $U(t,t_1,0) = U_0^{\dagger}(t,t_1)U(t,0)U_0^{\dagger}(t_1,0)$ $(t > t_1)$ and $U_0(t,t_1)$ denotes the evolution operator of the undriven system. From Eqs. (28) and (29) one obtains, after some mathematical manipulations, that

$$U(t,t_1,0) = \hat{\mathrm{T}}\exp\left[-\frac{i}{\hbar}\int_{-t_1}^{t-t_1} e^{iH^{(0)}t'/\hbar}V(t'+t_1)e^{-iH^{(0)}t'/\hbar}dt'\right] \ , \tag{30}$$

where $\hat{\mathrm{T}}$ is the time ordering operator. For the case of a short interaction time the time ordering in Eq. (30) can be neglected. This approximation is equivalent to the Magnus expansion of the exponential up to a first order.[22] Then for the case of a short pulse, the propagator can be approximated as

$$U(t,t_1,0) = \exp\left[-\frac{i}{\hbar}\int_{-t_1}^{t-t_1} e^{iH^{(0)}t'/\hbar}V(t'+t_1)e^{-iH^{(0)}t'/\hbar}dt'\right] \ . \tag{31}$$

Taking into account the identity

$$e^{-A}Be^{A} = B + [B, A] + \frac{1}{2!}[(B, A), A] + \dots \quad , \tag{32}$$

one finds from Eq. (31) the following expansion for the propagator

$$U(t, t_1, 0) = \exp\left[-\frac{i}{\hbar}V_0 + \frac{i}{\hbar^2}[V_1, iH^{(0)}] + \frac{i}{2!\hbar^3}\{[V_2, H^{(0)}], H^{(0)}\} + \dots\right] \quad , \tag{33}$$

where

$$V_n = \int_0^t (t' - t_1)^n V(t')dt' \quad , \quad n = 0, 1, 2, \dots \quad t > t_1 \quad . \tag{34}$$

For the case of our interest here the interaction potential $V(t)$ has the general form $V(t) = \mathbf{r}.\mathbf{e}Fa(t - t_1)$, with $\mathbf{r}$ deuoting the spatial coordinate of the carrier, $\mathbf{e}$ the polarization vector of the external field, F the peak amplitude of the field, and $a(t - t_1)$ the envelope of the pulse. We note that since the pulse is strongly peaked around t_1, the lower and upper integration limits in Eq. (34) can be set to $-\infty$ and ∞, respectively. In the case the pulse has a Gaussian-like shape, i.e., $a(t - t_1) = \exp[-(t - t_1)^2/(2\sigma^2)]$ one obtains

$$V_0 = \mathbf{r}.\mathbf{e}F\sigma\sqrt{2\pi} \quad , \quad V_1 = 0 \quad , \quad V_2 = V_0\sigma^2 \quad . \tag{35}$$

On the other hand, $H^{(0)}/\hbar \sim \tau_c^{-1}$ (here τ_c refers to the characteristic time of the unperturbed system). Therefore, if $\sigma/\tau_c \ll 1$ (i.e., if the duration of the pulse is much smaller than the characteristic time of the undriven system) then the first term of the expansion in Eq. (33) suffices. In such a case the propagator in Eq. (33) can be written as

$$U(t, t_1, 0) = \exp\left[\frac{i}{\hbar}\mathbf{r}.\mathbf{p}\right] \quad , \tag{36}$$

where

$$\mathbf{p} = -\mathbf{e}F\int_{-\infty}^{\infty} a(t')dt' \tag{37}$$

is determined by the area of the pulse. The evolution operator in Eq. (29) takes then the following form

$$U(t, t_0) = U_0(t, t_1)e^{\frac{i}{\hbar}\mathbf{r}.\mathbf{p}}U_0(t_1, 0) \quad . \tag{38}$$

This equation reveals the essence of the impulsive approximation: the action of the pulse can be interpreted as an instantaneous *kick* that transfers a momentum $\mathbf{p}$ to the system. In particular, one deduces from Eq. (38) that the wave function of the system right before ($t = t_1^-$) and just after ($t = t_1^+$) the application of the pulse satisfies the following matching condition

$$\Psi(\mathbf{r}, t_1^+) = e^{\frac{i}{\hbar}\mathbf{r}.\mathbf{p}}\Psi(\mathbf{r}, t_1^-) \quad . \tag{39}$$

REFERENCES

1. R. R. Jones, D. You, and P. H. Bucksbaum, Phys. Rev. Lett. **70**, 1236 (1993); *ibid* **76**, 3927 (1996).
2. A. Bugacov, B. Piraux, M. Pont, and R. Shakeshaft, Phys. Rev. A **51**, 1490 (1995); **51**, 4877 (1995); F. Robicheaux, Phys. Rev. A **60**, 431 (1999).
3. C.L. Stokely, F.B. Dunning, C.O. Reinhold, and A.K. Pattanayak, Phys. Rev. A **65**, 021405 (2002).
4. R. Bavli and H. Metiu, Phys. Rev. Lett. **69**, 229 (1998); Phys. Rev. A **47**, 3299 (1993).
5. Y. Imry, *Introduction to mesoscopic physics*, Scnd. edition (University press, Oxford, 2002).
6. M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. A **96**, 365 (1983).
7. R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985).
8. H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih, Phys. Rev. B **37**, 6050 (1988).
9. J. F. Weisz, R. Kishore, and F. V. Kusmartsev, Phys. Rev. B **49**, 8126 (1994).
10. W. C. Tan and J. C. Inkson, Phys. Rev. B **60**, 5626 (1999).

11. D. Loss and P. Goldbart, Phys. Rev. B **43**, 13762 (1991).
12. S. A. Washburn and R. A. Webb, Add. Phys. **35**, 375 (1986).
13. K. B. Efetov, Phys. Rev. Lett. **66**, 2794 (1991).
14. V. E. Kravtsov and V. I. Yudson, Phys. Rev. Lett. **70**, 210 (1993).
15. O. L. Chalaev and V. E. Kravtsov, Phys. Rev. Lett. **89**, 176601 (2002).
16. P. Kopietz and A. Völker, Eur. Phys. J. B **3**, 397 (1998).
17. M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 245321 (2002).
18. K. Yakubo and J. Ohe, Physica E **18**, 97 (2003).
19. G. M. Genking and G. A. Vugalter, Phys. Lett. A **189**, 415 (1994).
20. G. Bastard *Wave mechanics applied to semiconductor heterostructures* (France: Les éditions de physique, 1996).
21. R. Heather and H. Metiu, J. Chem. Phys. **86**, 9 (1987).
22. N. E. Henriksen, Chem. Phys. Lett. **312**, 196 (1999).
23. T. Swahn, E. N. Bogachek, Y. M. Galperin, M. Jonson, and R. I. Shekhter, Phys. Rev. Lett. **73**, 162 (1994).
24. D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).
25. J. M. Ziman, *Principles of the Theory of Solids*, Scnd. edition (University press, Cambridge, 1998).
26. W. Rabaud, L. Saminadayar, D. Mailly, K. Hasselbach, A. Benoit, and B. Etienne, Phys. Rev. Lett. **86**, 3124 (2001).