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Classical description of current-induced spin-transfer torque in multilayer structures

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We develop a classical description of the current-induced torque due to spin transfer in a layered system consisting of two ferromagnetic films separated by a nonmagnetic layer. The description is based on the classical equations for time-space evolution of the macroscopic magnetization. It is assumed that the perpendicular component of the nonequilibrium magnetization relaxes very fast in ferromagnetic films. Such a fast relaxation is due to a strong exchange field. Accordingly, the perpendicular component is totally absorbed at the interfaces giving rise to the torque. The longitudinal component, on the other hand, decays on a much longer distance defined by the spin diffusion length. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1836861]

I. INTRODUCTION

Magnetic configuration of a system composed of two ferromagnetic metallic films separated by a nonmagnetic metallic layer has a significant impact on the electric current flowing through the system. When this configuration varies from an antiparallel alignment to a parallel one, the electric current increases significantly (at a constant bias voltage).[1,2] It is also quite natural to expect that the electric current flowing through such a system will have an influence on its magnetic state, too. The existence of a torque, which a spin polarized current exerts on a magnetic layer was predicted theoretically by Berger[3] and Slonczewski.[4] At suitable conditions, such a torque may cause rotation of the magnetic moment of a certain film. Indeed, current induced switching between different magnetic configurations was observed recently in a number of experiments.[5–8] For instance, Katine *et al.*[7] observed current induced switching in a structure consisting of two Co films (of different thicknesses) separated by a Cu layer, and with two Cu leads attached to the system. They have shown that the switching occurs when the electric current exceeds a certain critical value. Moreover, they have also shown that the switching depends on the current direction—for a certain bias polarization the switching is to the parallel configuration whereas for the opposite bias polarization the system switches to the antiparallel state.

The switching phenomenon was described by Heide *et al.*[9] in terms of an additional nonequilibrium current-induced exchange interaction between the magnetic films. The basic properties of such nonequilibrium exchange interaction are significantly different from those of the usual indirect Ruderman-Kittel-Kasuya-Yosida (RKKY) exchange coupling. As the sign of the RKKY coupling parameter oscillates with the thickness of the nonmagnetic spacer layer, the relevant parameter describing nonequilibrium exchange interaction varies monotonously with this thickness. In addition, it changes sign when the direction of the flowing current is changed and vanishes in symmetrical situation (both magnetic films are equivalent).

The physical origin of the current induced switching is still under discussion in the relevant literature, and several theoretical models have been proposed to account for the phenomenon.[9–15] Roughly speaking, the current driven switching is a result of a torque which spin polarized current exerts on a particular film. Since electrons of opposite spin orientations are reflected back (or transmitted) with spin dependent reflection (transmission) coefficients, the angular momentum transmitted from the electron system to the magnetic layer produces a torque which is responsible for the magnetic switching.

In a recent paper Zhang *et al.*[16] used a quasiclassical model to study the current induced switching phenomenon. They have shown that the key point is the spin accumulation associated with spin dependent transmission/reflection at the interfaces. More specifically, they have shown that it is the transversal component of the spin accumulation that contributes to the torque. This transversal component relaxes very fast due to exchange interaction between the conduction electrons and local moments. On the other hand, the longitudinal part of spin accumulation does not influence the switching mechanism, because it does not exert any torque on the local magnetization. Moreover, the longitudinal spin accumulation relaxes much more slowly than the transversal one. Thus, the torque is exerted on a very thin part of a magnetic film at its surface.[16,17] This was supported by microscopic quantum-mechanical considerations[18] and also assumed in other macroscopic theoretical descriptions.[19–23]

In this paper we develop a classical description of the switching phenomenon. The stationary macroscopic spin density and the charge/spin currents are described by the classical diffusion equations. We assume that the perpendicular component of the spin current is absorbed at the very interface. This assumption allows us to derive some effective boundary conditions for the drops of the spin density and spin current at the interfaces. The macroscopic equations describing the charge and spin currents as well as the spin density inside the films are presented in Sec. II. The boundary conditions are derived in Sec. III. Numerical results for a structure with two magnetic films are presented and discussed in Sec. IV whereas final conclusions are in Sec. V.

II. CLASSICAL DESCRIPTION

Assume a charge current j_0 flowing along the axis x which is normal to the layers of a multilayered structure. The axes y and z of the coordinate system used in the description are then in the film plane. With each ferromagnetic layer we associate a local coordinate system, whose axis z is along the corresponding equilibrium spin density (local quantization axis).

In a nonequilibrium state, an inhomogeneous electron spin density (spin accumulation) $\mathbf{m}(x,t)$ can build up, which is assumed to be uniform in the film plane and to depend on the coordinate x. The classical equations for the space-time evolution of the electron spin density $\mathbf{m}(x,t)$ in a magnetic film can be written in the form

$$\frac{\partial m_z}{\partial t} + \frac{\partial J_z}{\partial x} = -\frac{\delta m_z}{\tau_l}, \tag{1}$$

$$\frac{\partial m_{x,y}}{\partial t} + \frac{\partial J_{x,y}}{\partial x} = -\frac{\delta m_{x,y}}{\tau_t}, \tag{2}$$

where the spin current density $\mathbf{J}(x,t)$ is given by

$$J_z = -D_l\frac{\partial m_z}{\partial x} + b_z, \tag{3}$$

$$J_{x,y} = -D_t\frac{\partial m_{x,y}}{\partial x} + b_{x,y}. \tag{4}$$

Here, D_l and D_t are the longitudinal and transversal components of the spin diffusion tensor, τ_l and τ_t are the corresponding relaxation times, whereas δm_z and $\delta m_{x,y}$ are the longitudinal and transversal deviations of the spin density from the equilibrium value $\mathbf{m}^{(0)}=(0,0,m^{(0)})$. It should be noted that usually $\tau_t \ll \tau_l$ in ferromagnetic metals, so that the transverse components $\delta m_{x,y}$ vanish very quickly and can be neglected. The second terms on the right hand sides of Eqs. (3) and (4) are the drag components of the spin current in the presence of a charge current j_0 which can be written as

$$b_z = \frac{j_0}{e}\frac{n_0(\tau_\uparrow - \tau_\downarrow) + m_z(\tau_\uparrow + \tau_\downarrow)}{n_0(\tau_\uparrow + \tau_\downarrow) + m_z(\tau_\uparrow - \tau_\downarrow)}, \tag{5}$$

$$b_{x,y} = \frac{j_0}{e}\frac{m_{x,y}}{n_0}, \tag{6}$$

where $\tau_\uparrow$ and $\tau_\downarrow$ are the momentum relaxation times for spin-up (majority) and spin-down (minority) electrons, respectively, n_0 is the total concentration of electrons, and e is the electron charge $(e<0)$.

If we adopt the diffusive approximation for the distribution of spin-up and spin-down electrons, valid at distances much longer than the electron mean free paths, the longitudinal spin diffusion coefficient in the ferromagnet D_l can be presented as $D_l=(D_\uparrow+D_\downarrow)/2$. Here $D_\uparrow=v_\uparrow^2\tau_\uparrow/3$ and $D_\downarrow=v_\downarrow^2\tau_\downarrow/3$ are the diffusion coefficients of spin-up and spin-down electrons, respectively.

For clarity of notation, the (x,t) arguments of the electron spin density $\mathbf{m}$, $\delta\mathbf{m}$, and of the spin current $\mathbf{J}$ (and consequently also of $\mathbf{b}$) have been omitted and will be restored in the following only when necessary. Formula (6) for $b_{x,y}$ indicates that $J_{x,y}$ vanish when the transversal components $m_{x,y}$ of the magnetization are equal to zero. This is not true for J_z since b_z generally does not vanish for $m_z=0$. The corresponding equations for a nonmagnetic metal have similar form, but with $\tau_l=\tau_t=\tau$, $D_l=D_t=D$, $\tau_\uparrow=\tau_\downarrow=\tau_0$, and $\mathbf{m}^{(0)}=0$. In the following we will consider only stationary situations of the above equations.

Let us consider an interface between nonmagnetic and ferromagnetic metals labeled as (1) and (2), respectively. Assume that the static equilibrium spin density $\mathbf{M}$ in the ferromagnet is parallel to the interface, and is along the axis z. The boundary conditions at the interface are related to the properties of contacting materials and to the specific properties of the interface. Generally, one can write the boundary conditions in the following form:

$$\mathbf{m}^{(1)} = \mathbf{m}^{(2)} + \Delta\mathbf{m}, \tag{7}$$

$$\mathbf{J}^{(1)} = \mathbf{J}^{(2)} + \Delta\mathbf{J}, \tag{8}$$

where $\Delta\mathbf{m}$ and $\Delta\mathbf{J}$ are the drops of the spin density and spin currents at the interface, respectively. In the absence of spin-flip scattering at the interface, the longitudinal component of the spin current J_z is conserved, and consequently $\Delta J_z=0$.

III. TRANSMISSION THROUGH THE INTERFACE

To determine the parameters for the boundary conditions at the interface we need to use a microscopic description of the transmission through the interface. Let us consider a contact between nonmagnetic [$x<0$, labeled as (1)] and magnetic [$x>0$, labeled as (2)] metals, with no intrinsic spin flip processes at the interface. As above, we assume the quantization axis z along the magnetization vector $\mathbf{M}$ of the magnetic metal. The amplitudes of waves corresponding to the wave vector $\mathbf{k}$ and propagating in opposite directions along the axis x (labeled with "$>$" for $k_x>0$ and "$<$" for $k_x<0$) are related *via* the following equations:

$$b_\sigma^> = t_\sigma a_\sigma^> + r_\sigma b_\sigma^<, \tag{9}$$

$$a_\sigma^< = t_\sigma^* b_\sigma^< + r_\sigma^* a_\sigma^>, \tag{10}$$

where a and b describe the amplitudes of waves propagating in metals (1) and (2), respectively, whereas t_σ and r_σ are the transmission and reflection amplitudes for $\sigma=\uparrow,\downarrow$. For simplicity we assume a perfect interface, when the in-plane component of the wave vector $\mathbf{k}$ is conserved during reflection (transmission), and for clarity of notation we dropped the index $\mathbf{k}$ labeling the states.

The distribution functions can be calculated as the quantum mechanical averages of the corresponding wave function amplitudes (density matrix[24])

$$f^{>}_{\sigma\sigma'} = \langle a^{>*}_{\sigma} a^{>}_{\sigma'} \rangle, \quad g^{>}_{\sigma\sigma'} = \langle b^{>*}_{\sigma} b^{>}_{\sigma'} \rangle, \tag{11}$$

and similarly for amplitudes labeled with $<$. The distribution functions $f_{\sigma\sigma'}$ and $g_{\sigma\sigma'}$ refer to nonmagnetic metal (1) and ferromagnet (2), respectively. Using Eqs. (9) and (10), one obtains the following equations for the diagonal in spin components of the distribution functions:

$$g^{>}_{\sigma\sigma} = T_{\sigma} f^{>}_{\sigma\sigma} + R_{\sigma} g^{<}_{\sigma\sigma}, \tag{12}$$

$$f^{<}_{\sigma\sigma} = T_{\sigma} g^{<}_{\sigma\sigma} + R_{\sigma} f^{>}_{\sigma\sigma}, \tag{13}$$

where $R_{\sigma}=|r_{\sigma}|^2$ and $T_{\sigma}=|t_{\sigma}|^2$ are the reflection and transmission coefficients, respectively. The off-diagonal in spin component of the distribution function is not vanishing only for the nonmagnetic metal. In the ferromagnet the average (11) vanishes for $\sigma \neq \sigma'$ due to the different periodicity of the wave function oscillations for spin-up and spin-down electrons.[3,15] Thus, for $\sigma \neq \sigma'$ one obtains

$$f^{<}_{\sigma\sigma'} = \tilde{R} f^{>}_{\sigma\sigma'}, \quad g^{<}_{\sigma\sigma'} = g^{>}_{\sigma\sigma'} = 0, \tag{14}$$

where $\tilde{R}=r^{*}_{\sigma} r_{\sigma'}$. Equations (12)–(14) have the form of kinetic equations for the distribution functions at the interface.

It is convenient to use the following representation for the distribution functions $f^{>}_{\sigma\sigma'}$

$$\check{f}^{>} = \begin{pmatrix} f^{>}_{\uparrow\uparrow} & f^{>}_{\uparrow\downarrow} \\ f^{>}_{\downarrow\uparrow} & f^{>}_{\downarrow\downarrow} \end{pmatrix} = f^{>}_0 \check{1} + f^{>}_x \check{\sigma}_x + f^{>}_y \check{\sigma}_y + f^{>}_z \check{\sigma}_z, \tag{15}$$

and similar ones for $f^{<}_{\sigma\sigma'}$, $g^{>}_{\sigma\sigma'}$, and $g^{<}_{\sigma\sigma'}$. In Eq. (15) $\check{1}$ is the 2×2 unit matrix, whereas $\check{\sigma}_x$, $\check{\sigma}_y$, and $\check{\sigma}_z$ are the Pauli matrices. From Eq. (15) follows that $f^{>}_x = \mathrm{Re}\, f^{>}_{\uparrow\downarrow}$, $f^{>}_y = \mathrm{Im}\, f^{>}_{\uparrow\downarrow}$, $f^{>}_z = (f^{>}_{\uparrow\uparrow} - f^{>}_{\downarrow\downarrow})/2$, and $f^{>}_0 = (f^{>}_{\uparrow\uparrow} + f^{>}_{\downarrow\downarrow})/2$. Similar formulae also hold for the other distribution functions.

The spin density near the interface in metal (1) is then given by the formula

$$\mathbf{m} = \sum_{\mathbf{k}} \mathrm{Tr}\{\check{\boldsymbol{\sigma}}(\check{f}^{>} + \check{f}^{<})\} = 2\sum_{\mathbf{k}} (\mathbf{f}^{>} + \mathbf{f}^{<}), \tag{16}$$

where $\mathbf{f}^{>} \equiv (f^{>}_x, f^{>}_y, f^{>}_x)$ (similarly for $\mathbf{f}^{<}$), and the sum runs over all filled states with energies below the Fermi energy ε_F. Similar formulas also hold for the spin density near the interface in the metal (2).

Taking Eqs. (12)–(14) into account one can write

$$m^{(1)}_x = 2\sum_{\mathbf{k}} [(1 + \mathrm{Re}\, \tilde{R}) f^{>}_x - \mathrm{Im}\, \tilde{R} f^{>}_y], \tag{17}$$

$$m^{(1)}_y = 2\sum_{\mathbf{k}} [(1 + \mathrm{Re}\, \tilde{R}) f^{>}_y + \mathrm{Im}\, \tilde{R} f^{>}_x], \tag{18}$$

$$m^{(1)}_z = \sum_{\mathbf{k}} [(2 + R_{\uparrow} + R_{\downarrow}) f^{>}_z + (R_{\uparrow} - R_{\downarrow}) f^{>}_0 + (T_{\uparrow} + T_{\downarrow}) g^{<}_z + (T_{\uparrow} - T_{\downarrow}) g^{<}_0], \tag{19}$$

for layer (1), and

$$m^{(2)}_{x,y} = 0, \tag{20}$$

$$m^{(2)}_z = \sum_{\mathbf{k}} [(2 + R_{\uparrow} + R_{\downarrow}) g^{<}_z + (R_{\uparrow} - R_{\downarrow}) g^{<}_0 + (T_{\uparrow} + T_{\downarrow}) f^{>}_z + (T_{\uparrow} - T_{\downarrow}) f^{>}_0], \tag{21}$$

for the second layer (2).

Thus, the spin density drop across the interface is given by

$$\Delta m_x = 2\sum_{\mathbf{k}} [(1 + \mathrm{Re}\, \tilde{R}) f^{>}_x - \mathrm{Im}\, \tilde{R} f^{>}_y], \tag{22}$$

$$\Delta m_y = 2\sum_{\mathbf{k}} [(1 + \mathrm{Re}\, \tilde{R}) f^{>}_y + \mathrm{Im}\, \tilde{R} f^{>}_x], \tag{23}$$

$$\Delta m_z = 2\sum_{\mathbf{k}} [(R_{\uparrow} + R_{\downarrow})(f^{>}_z - g^{<}_z) + (R_{\uparrow} - R_{\downarrow})(f^{>}_0 - g^{<}_0)]. \tag{24}$$

The spin current components at the interface can be calculated in a similar way, and one finds

$$J^{(1)}_x = 2\sum_{\mathbf{k}} v_x [(1 - \mathrm{Re}\, \tilde{R}) f^{>}_x + \mathrm{Im}\, \tilde{R} f^{>}_y], \tag{25}$$

$$J^{(1)}_y = 2\sum_{\mathbf{k}} v_x [(1 - \mathrm{Re}\, \tilde{R}) f^{>}_y - \mathrm{Im}\, \tilde{R} f^{>}_x], \tag{26}$$

and

$$J^{(2)}_{x,y} = 0, \tag{27}$$

$$J^{(1)}_z = J^{(2)}_z = \sum_{\mathbf{k}} v_x [(T_{\uparrow} + T_{\downarrow})(f^{>}_z - g^{<}_z) + (T_{\uparrow} - T_{\downarrow})(f^{>}_0 - g^{<}_0)], \tag{28}$$

where v_x is the electron velocity along the axis x (generally v_x depends on the vave vector $\mathbf{k}$).

Thus, the drops of x and y components of the spin currents are

$$\Delta J_x = 2\sum_{\mathbf{k}} v_x [(1 - \mathrm{Re}\, \tilde{R}) f^{>}_x + \mathrm{Im}\, \tilde{R} f^{>}_y], \tag{29}$$

$$\Delta J_y = 2\sum_{\mathbf{k}} v_x [(1 - \mathrm{Re}\, \tilde{R}) f^{>}_y - \mathrm{Im}\, \tilde{R} f^{>}_x]. \tag{30}$$

The z components of the spin current and of the spin density drop include the distribution functions $f^{>}_0$ and $g^{<}_0$, which can be eliminated by calculating the charge current on both sides of the interface. This current, in turn, has to be the same as the charge current in the bulk. From calculations similar to those for the spin currents it follows that the charge current at the interfaces is

$$j_0^{(1)} = j_0^{(2)} = e\sum_{\mathbf{k}} v_x[(T_\uparrow - T_\downarrow)(f_z^> - g_z^<) + (T_\uparrow + T_\downarrow)(f_0^> - g_0^<)]. \tag{31}$$

The drops in spin currents at the interface can be related to the drops in spin density. However, to get simple analytical expressions we need to simplify the formulation. First, in the above equations we replace the reflection and transmission coefficients by their average values and take them out of the summations. Second, we replace the electron velocity along the axis x by its average value v (the average is over half of the Fermi sphere, $v_x > 0$). This allows us to take these quantities out of the summations as well and write the boundary conditions in the form

$$(1 - \tilde{R}\tilde{R}^*)\Delta J_x - 2\,\mathrm{Im}\,\tilde{R}\Delta J_y = v[(1 - \mathrm{Re}\,\tilde{R})^2 + (\mathrm{Im}\,\tilde{R})^2]\Delta m_x, \tag{32}$$

$$(1 - \tilde{R}\tilde{R}^*)\Delta J_y + 2\,\mathrm{Im}\,\tilde{R}\Delta J_x = v[(1 - \mathrm{Re}\,\tilde{R})^2 + (\mathrm{Im}\,\tilde{R})^2]\Delta m_y, \tag{33}$$

$$J_z^{(1,2)}\left(\frac{R_\uparrow + R_\downarrow}{v} + \frac{R_\uparrow}{v_\uparrow} + \frac{R_\downarrow}{v_\downarrow}\right) = \Delta m_z(T_\uparrow + T_\downarrow) - \frac{j_0}{e}\left(\frac{R_\uparrow - R_\downarrow}{v} + \frac{R_\uparrow}{v_\uparrow} - \frac{R_\downarrow}{v_\downarrow}\right), \tag{34}$$

where v, $v_\uparrow$, and $v_\downarrow$ are the average velocities of electrons in the nonmagnetic metal and the ferromagnet for spin-up and spin-down electrons, respectively. In planar systems, the average velocities are equal to half of the corresponding Fermi velocities, whereas in a one-dimensional case they are equal to the Fermi velocities.

IV. FOUR-LAYER STRUCTURE

Consider now a four-layer structure consisting of a thick magnetic layer (e.g., Co), thin nonmagnetic layer (e.g., Cu) followed by a thin magnetic layer (Co), and then again a thick nonmagnetic layer (Cu), as presented schematically in Fig. 1. For simplicity, we assume the outermost magnetic and nonmagnetic layers to be infinitely thick. Apart form this, we assume the local coordinate system of the thick magnetic layer (labeled with the index 1) as the global one for the whole structure. In turn, the equilibrium spin density of the thin magnetic layer (3) is assumed to be rotated by the angle θ around the axis x (see Fig. 1). Accordingly, the magnetization vector of the layer (3) is also parallel to the interface.

For the numerical calculations we assume $\tilde{R}$ is real. Such an assumption is not far from reality because the imaginary part of $\tilde{R}$ is usually small. Under this assumption the x components of the spin density and current in each layer then vanishes exactly. Thus, we have to solve the equations only for the y and z components. The boundary conditions (32) and (33) then reduce to

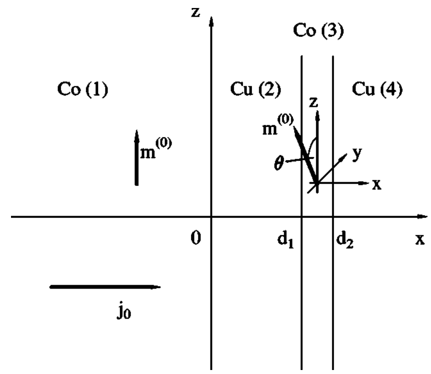

FIG. 1. Schematic structure consisting of a thick ferromagnetic film (Co), nonmagnetic metallic layer (Cu), a thin ferromagnetic layer (Co) followed by a thick nonmagnetic film (Cu). The arrows indicate orientation of the equilibrium spin density in the ferromagnetic films.

$$\Delta J_y = v\frac{(1 - \tilde{R})}{1 + \tilde{R}}\Delta m_y. \tag{35}$$

The general solution of Eqs. (1)–(4) for the magnetic layer (1), with a nonequilibrium magnetization $\delta m_i^{(1)}$ decaying for $x \to -\infty$, has the following form (we assume that $\delta m_z / m^{(0)} \ll 1$ and use a corresponding expansion up to the linear term, $b_l \simeq b_l^{(0)} + b_l^{(1)}\delta m_z$)

$$m_y^{(1)} = 0, \quad \delta m_z^{(1)} = C_1 e^{\mu_1 x}, \tag{36}$$

$$J_y^{(1)} = 0, \quad J_z^{(1)} = (-D_l\mu_1 + b_l^{(1)})C_1 e^{\mu_1 x} + b_l^{(0)}, \tag{37}$$

where

$$\mu_{1,2} = \frac{b_l^{(1)}}{2D_l} \pm \left(\frac{(b_l^{(1)})^2}{4D_l^2} + \frac{1}{D_l\tau_l}\right)^{1/2}, \tag{38}$$

$$b_l^{(0)} = \frac{j_0}{e}\frac{n_0(\tau_\uparrow - \tau_\downarrow) + m^{(0)}(\tau_\uparrow + \tau_\downarrow)}{n_0(\tau_\uparrow + \tau_\downarrow) + m^{(0)}(\tau_\uparrow - \tau_\downarrow)} \tag{39}$$

$$b_l^{(1)} = \frac{j_0}{e}\frac{4n_0\tau_\uparrow\tau_\downarrow}{[n_0(\tau_\uparrow + \tau_\downarrow) + m^{(0)}(\tau_\uparrow - \tau_\downarrow)]^2}, \tag{40}$$

and C_1 is a constant. Although Eq. (36) contains only μ_1, we define in Eq. (38) both μ_1 and μ_2 (the latter will be used later on).

The corresponding solutions of Eqs (1)–(4) for the nonmagnetic (2) layer is

$$\delta m_y^{(2)} = C_2 e^{\nu_1 x} + C_3 e^{\nu_2 x},$$

$$\delta m_z^{(2)} = C_4 e^{\nu_1 x} + C_5 e^{\nu_2 x}, \tag{41}$$

$$J_y^{(2)} = (-D\nu_1 + b)C_2 e^{\nu_1 x} + (-D\nu_2 + b)C_3 e^{\nu_2 x},$$

$$J_z^{(2)} = (-D\nu_1 + b)C_4 e^{\nu_1 x} + (-D\nu_2 + b)C_5 e^{\nu_2 x}, \tag{42}$$

where

$$\nu_{1,2} = \frac{b}{2D} \pm \left(\frac{b^2}{4D^2} + \frac{1}{D\tau}\right)^{1/2}. \tag{43}$$

Equilibrium spin density of the thin magnetic layer (3) is rotated by the angle θ around the axis x. We find first the corresponding solution in the local coordinate system and then transform it to the global one. Following this one obtains

$$\delta m_y^{(3)} = -\sin\theta(C_6 e^{\mu_1 x} + C_7 e^{\mu_2 x}),$$

$$\delta m_z^{(3)} = \cos\theta(C_6 e^{\mu_1 x} + C_7 e^{\mu_2 x}), \tag{44}$$

$$J_y^{(3)} = -\sin\theta[(-D_l\mu_1 + b_l^{(1)})C_6 e^{\mu_1 x} + (-D_l\mu_2 + b_l^{(1)})C_7 e^{\mu_2 x} + b_l^{(0)}], \tag{45}$$

$$J_z^{(3)} = \cos\theta[(-D_l\mu_1 + b_l^{(1)})C_6 e^{\mu_1 x} + (-D_l\mu_2 + b_l^{(1)})C_7 e^{\mu_2 x} + b_l^{(0)}]. \tag{46}$$

Finally, the solutions for Cu (4) layer, which vanish at $x \to 0$, are

$$\delta m_y^{(4)} = -\sin\theta C_8 e^{\nu_2 x}, \quad \delta m_z^{(4)} = \cos\theta C_8 e^{\nu_2 x}, \tag{47}$$

$$J_y^{(4)} = -\sin\theta(-D\nu_2 + b)C_8 e^{\nu_2 x},$$

$$J_z^{(4)} = \cos\theta(-D\nu_2 + b)C_8 e^{\nu_2 x}. \tag{48}$$

Here, we explicitly took into account the vanishing transverse components of the spin density and of the spin current in the thick nonmagnetic layer.

The boundary conditions at each interface determine completely the coefficients in the above general solutions for each layer of the structure. We use the conditions in the form of Eqs. (34) and (35), and the conservation of J_z at each of the interfaces (for the local quantization axis determined by the magnetization of the adjacent ferromagnet). This reduces the problem to a system of linear equations for the coefficients C_1 to C_8, which can be solved numerically.

The explicit form of the equations determining the coefficients C_1 to C_8 is

$$(D\nu_1 - b + p)C_2 + (D\nu_2 - b + p)C_3 = 0, \tag{49}$$

$$(D_l\mu_1 - b_l^{(1)})C_1 - (D\nu_1 - b)C_4 - (D\nu_2 - b)C_5 = b_l^{(0)}, \tag{50}$$

$$(D_l\mu_1 - b_l^{(1)} + p_1)C_1 - p_1 C_4 - p_1 C_5 = b_l^{(0)} + p_2, \tag{51}$$

$$(D\nu_1 - b + p)C_2 e^{\nu_1 d_1}\cos\theta + (D\nu_2 - b + p)C_3 e^{\nu_2 d_1}\cos\theta + (D\nu_1 - b + p)C_4 e^{\nu_1 d_1}\sin\theta + (D\nu_2 - b + p)C_5 e^{\nu_2 d_1}\sin\theta = 0, \tag{52}$$

$$(D\nu_1 - b)C_2 e^{\nu_1 d_1}\sin\theta + (D\nu_2 - b)C_3 e^{\nu_2 d_1}\sin\theta - (D\nu_1 - b)C_4 e^{\nu_1 d_1}\cos\theta - (D\nu_2 - b)C_5 e^{\nu_2 d_1}\cos\theta + (D_l\mu_1 - b_l^{(1)})C_6 e^{\mu_1 d_1} + (D_l\mu_2 - b_l^{(1)})C_7 e^{\mu_2 d_1} = b_l^{(0)}, \tag{53}$$

$$(D_l\mu_1 - b_l^{(1)} - p_1)C_6 e^{\mu_1 d_1} + (D_l\mu_2 - b_l^{(1)} - p_1)C_7 e^{\mu_2 d_1} - p_1 C_2 e^{\nu_1 d_1}\sin\theta - p_1 C_3 e^{\nu_2 d_1}\sin\theta + p_1 C_4 e^{\nu_1 d_1}\cos\theta + p_1 C_5 e^{\nu_2 d_1}\cos\theta = b_l^{(0)} + p_2, \tag{54}$$

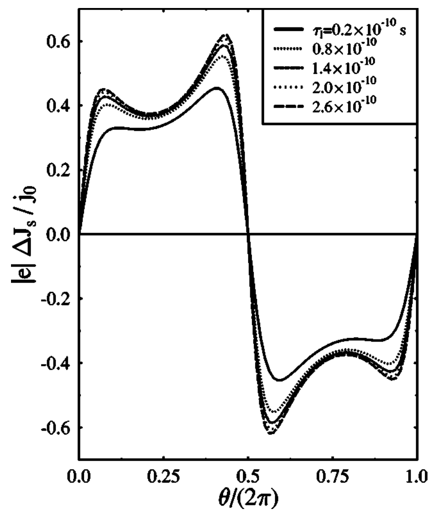

FIG. 2. Transferred torque as a function of the angle θ for $\tau = 10^{-10}$ s and for different values of τ_l.

$$(D_l\mu_1 - b_l^{(1)})C_6 e^{\mu_1 d_2} + (D_l\mu_2 - b_l^{(1)})C_7 e^{\mu_2 d_2} - (D\nu_2 - b)C_8 e^{\nu_2 d_2} = b_l^{(0)}, \tag{55}$$

$$(D\nu_2 - b - p_1)C_8 e^{\nu_2 d_2} + p_1 C_6 e^{\mu_1 d_2} + p_1 C_7 e^{\mu_2 d_2} = p_2, \tag{56}$$

where we defined

$$p = v\frac{1 - \tilde{R}}{1 + \tilde{R}}, \tag{57}$$

$$p_1 = (T_\uparrow + T_\downarrow)\left(\frac{R_\uparrow + R_\downarrow}{v} + \frac{R_\uparrow}{v_\uparrow} + \frac{R_\downarrow}{v_\downarrow}\right)^{-1}, \tag{58}$$

$$p_2 = \frac{j_0}{e}\left(\frac{R_\uparrow - R_\downarrow}{v} + \frac{R_\uparrow}{v_\uparrow} - \frac{R_\downarrow}{v_\downarrow}\right)\left(\frac{R_\uparrow + R_\downarrow}{v} + \frac{R_\uparrow}{v_\uparrow} + \frac{R_\downarrow}{v_\downarrow}\right)^{-1}. \tag{59}$$

We take the following values of the parameters for the magnetic (Co) and nonmagnetic (Cu) layers: $v = 3/4 v_F$ (the same for all mean velocities, defined as $v = \int_0^{k_F}(\hbar k/m)k^2 dk / \int_0^{k_F} k^2 dk$), $v_F = 3.7\times 10^7$ cm/s, $v_{F\uparrow} = 3.74\times 10^7$ cm/s, $v_{F\downarrow} = 2.9\times 10^7$ cm/s (corresponding to the Fermi wave vectors $k_F \simeq 1.28\times 10^8$ cm^{-1}, $k_{F\uparrow} \simeq 1.3\times 10^8$ cm^{-1}, and $k_{F\downarrow} \simeq 1.0\times 10^8$ cm^{-1} in Cu and Co, respectively), electron effective mass $m = 4m_0$, $D_{l\mathrm{Co}} = 15.7$ cm^2 s^{-1}, $D_{\mathrm{Cu}} = 41$ cm^2 s^{-1} (corresponding to the conductivity $\sigma_{\mathrm{Co}} = 1.8\times 10^5$ Ω^{-1} cm^{-1} and $\sigma_{\mathrm{Cu}} = 6.45\times 10^5$ Ω^{-1} cm^{-1}, $\tau_\uparrow = 2.1\times 10^{-13}$ s, $\tau_\downarrow = 0.53\times 10^{-13}$ s, $\tau_0 = 1.3\times 10^{-13}$ s, $m^{(0)} = 2\times 10^{22}$ cm^{-3}, $n_0 = 5.4\times 10^{22}$ cm^{-3}, $R_\downarrow = 0.017$, and $R_\uparrow = 10^{-4}$, $d_1 = 1.0\times 10^{-5}$ cm, and $d_2 = 1.5\times 10^{-5}$ cm.

The transversal component of the transferred spin current that acts as a torque on the Co (3) layer, has been calculated as

$$\tau = -\frac{\hbar}{2}[J_z^{(2)}(d_1)\sin\theta + J_y^{(2)}(d_1)\cos\theta]. \tag{60}$$

The angular variation of the torque normalized to the absolute values of the charge current is presented in Fig. 2 and in Fig. 3 for different values of the spin relaxation times

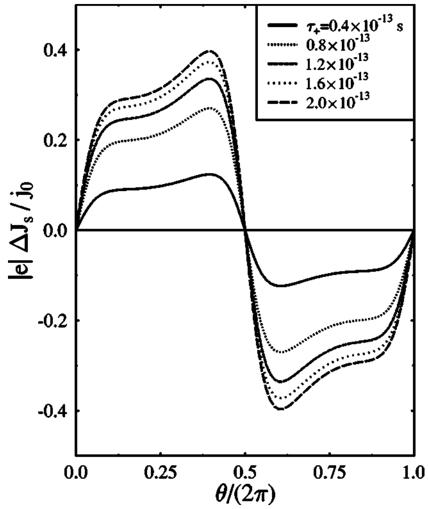

FIG. 3. Torque vs the angle θ for $\tau = 10^{-10}$ s, $\tau_l = 10^{-11}$ s, $\tau_{\downarrow} = 0.53 \times 10^{-13}$ s, and for different values of $\tau_{\uparrow}$.

in the ferromagnetic and nonmagnetic layers (for a positive current j_0). The torque is defined as positive when it tends to increase the angle and negative otherwise. First of all, spin torque vanishes for collinear configurations. However, the parallel configuration is unstable, since the torque for small deviations from the parallel configurations enhances the deviation. The stable configuration is the one with antiparallel magnetic moments. An interesting feature of the curves shown in Fig. 2 and Fig. 3 is the fast increase of the torque at small angles. In the case shown in Fig. 2, the torque has additionally a local minimum at a certain noncollinear configuration. Such a behavior seems to be consistent with some experimental observations which show that the magnetization rotation starts at a certain current value and then the current has to be increased to complete the rotation.[7,25]

V. CONCLUDING REMARKS

We have formulated a classical description of the magnetic switching in layered structures consisting of two nonequivalent magnetic layers separated by a nonmagnetic layer. The description assumes that the torque is due to the perpendicular component of spin current, which is totally absorbed at the interface when it enters the magnetic film. This effectively has been included into the boundary conditions. Thus, the torque can be calculated from diffusion type equations for the charge and spin currents inside the layers and from the boundary conditions at the interfaces. The numerical results obtained within this description are consistent with experimental observations.

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[1]M. N. Baibich et al., Phys. Rev. Lett. **61**, 2472 (1988).
[2]G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, Phys. Rev. B **39**, 4828 (1989).
[3]L. Berger, Phys. Rev. B **54**, 9353 (1996).
[4]J. C. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996); **195**, L261 (1999).
[5]E. B. Myers, D. C. Ralph, J. A. Katine, R. N. Louie, and R. A. Buhrman, Science **285**, 867 (1999).
[6]J. Z. Sun, J. Magn. Magn. Mater. **202**, 157 (1999).
[7]J. A. Katine, F. J. Albert, R. A. Buhrman, E. B. Myers, and D. C. Ralph, Phys. Rev. Lett. **84**, 3149 (2000).
[8]J. Grollier et al., Appl. Phys. Lett. **78**, 3663 (2001).
[9]C. Heide, P. E. Zilberman, and R. J. Elliott, Phys. Rev. B **63**, 064424 (2001); C. Heide, Phys. Rev. Lett. **87**, 197201 (2001).
[10]A. Brataas, Yu. V. Nazarov, and G. E. W. Bauer, Phys. Rev. Lett. **84**, 2481 (2000); D. H. Hernando, Yu. V. Nazarov, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **62**, 5700 (2000).
[11]X. Weintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph, Phys. Rev. B **62**, 12317 (2000).
[12]Yu. A. Bazaliy, B. A. Jones, and S. C. Zhang, cond-mat/0009034.
[13]J. E. Wegrowe, Phys. Rev. B **62**, 1067 (2000).
[14]J. Z. Sun, Phys. Rev. B **62**, 570 (2000).
[15]M. D. Stiles and A. Zangwill, J. Appl. Phys. **91**, 6812 (2002).
[16]S. Zhang, P. M. Levy, and A. Fert, Phys. Rev. Lett. **88**, 236601 (2002).
[17]A. Shpiro, P. M. Levy, and S. Zhang, Phys. Rev. B **67**, 104430 (2003).
[18]M. D. Stiles and A. Zangwill, Phys. Rev. B **66**, 014407 (2002).
[19]A. Brataas, Yu. V. Nazarov, and G. E. W. Bauer, Eur. Phys. J. B **22**, 99 (2001).
[20]Y. Tserkovnyak and A. Brataas, Phys. Rev. B **65**, 094517 (2002).
[21]K. Xia, P. J. Kelly, G. E. W. Bauer, A. Brataas, and I. Turek, Phys. Rev. B **65**, 220401 (2002).
[22]A. A. Kovalev, A. Brataas, and G. E. W. Bauer, Phys. Rev. B **66**, 224424 (2002).
[23]G. E. W. Bauer, Y. Tserkovnyak, D. Huertas-Hernando, and A. Brataas, Phys. Rev. B **67**, 094421 (2003).
[24]L. A. Falkovsky, Adv. Phys. **32**, 753 (1983).
[25]A. Fert et al., J. Magn. Magn. Mater. **272–276**, 1706 (2004).