

Magnetoresistance of a semiconducting magnetic wire with a domain wall

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We investigate theoretically the influence of the spin-orbit interaction of Rashba type on the magnetoresistance of a semiconducting ferromagnetic nanostructure with a laterally constrained domain wall. The domain wall is assumed sharp (on the scale of the Fermi wavelength of the charge carriers). It is shown that the magnetoresistance in such a case can be considerably large, which is in qualitative agreement with recent experimental observations. It is also shown that spin-orbit interaction may result in an increase of the magnetoresistance. The role of localization corrections is also briefly discussed.

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I. INTRODUCTION

Rapid progress in fabrication and measurement techniques of artificially sanitized ferromagnetic nanostructures revealed a variety of new phenomena.¹⁻⁴ For instance, in contrast to the bulk case, it has been found that the magnetoresistance associated with nanosize DWs can be very large.⁵⁻¹⁰ A notable example are the experiments on Ni microjunctions, which show that constrained DW formed at the contact of ferromagnetic wires results in a large electrical resistance, leading thus to a huge negative magnetoresistance.⁹ Further insight is provided by recent measurements of the magnetoresistance (found to be $\sim 2000\%$) in semiconducting magnetic nanoconstrictions.¹⁰ This latter example is particularly interesting insofar as the extent of DWs (i.e., the width L) formed in magnetic nanoconstrictions can be on the atomic scale¹¹ and considerably smaller than the Fermi wavelength of charge carriers. This situation may have important consequence as far as the influence of DW on the transport properties is concerned.

On the other hand, theoretical descriptions of the transport properties of DWs are mainly restricted to smooth DWs, typical for bulk or thin film ferromagnetic materials.¹²⁻¹⁶ Results of these studies indicate that electron scattering from smooth DWs is rather weak, and the spin of an electron propagating across the wall follows magnetization direction almost adiabatically. The contribution of smooth DWs to electrical resistance can be then calculated within the semiclassical approximation, and has been found to be either positive or negative—but in general it is rather small. We recall, however, that the condition for the applicability of the semiclassical approximation is $k_{F\uparrow(\downarrow)}L \gg 1$, where $k_{F\uparrow}$ and $k_{F\downarrow}$ are the Fermi wavevectors for the majority and minority electrons, respectively. This condition is fulfilled in bulk ferromagnets.

In contrast, for $k_{F\uparrow(\downarrow)}L \leq 1$, the semiclassical approximation is no longer valid and the scattering of electrons from the (sharp) DWs has to be considered strong. Therefore, vari-

ous attempts have been put forward to understand the influence of sharp DWs on transport properties. For instance, Tagirov *et al.*¹⁷ considered DWs in magnetic junctions as a potential barriers independent of the electron spin orientation. They concluded that the presence of DW results in a large magnetoresistance. Furthermore, ballistic electron transport through DWs was investigated numerically.¹⁸⁻²¹ Recently, the ballistic motion through a nanocontact has been studied by Zhuravlev *et al.*,²² who found a large magnetoresistance effect due to the presence of a nonmagnetic region within the constriction considered as a one-channel wire.

The one-dimensional model of a sharp DW has been considered in Ref. 23 in the limit of $k_{F\uparrow(\downarrow)}L \ll 1$. It has been shown there that the problem can be viewed as transmission through a spin-dependent barrier. This results in substantial magnetoresistance that increases when the spin polarization of electrons is enhanced. The largest magnetoresistance is thus expected for a fully spin-polarized electron gas.²⁴

A question which is still not yet addressed concerns the role of spin-orbit interaction in the scattering from a sharp DW. An analysis of this aspect is highly desirable in view of the relevance of spin-orbit interaction in spintronic devices, as evidenced by recent measurements.²⁵ Generally, the spin-orbit coupling can mix the spin channels, in addition to the mixing caused by the spin-dependent scattering from the DW. As demonstrated in this work, the presence of the spin-orbit interaction (of the Rashba type) results in an increase of the magnetoresistance due to DW. In the present work we also address briefly the role of localization corrections.

II. MODEL AND SCATTERING STATES

We consider a ferromagnetic narrow channel with a single magnetic DW. In the continuous model the spin density (magnetization) is a function of the coordinate z (along the channel), $\mathbf{M}(z) = [M_0 \sin \varphi(z), 0, M_0 \cos \varphi(z)]$, where $\varphi(z)$ varies continuously from zero to π for z changing from $z = -\infty$ to $z = +\infty$. Accordingly, the magnetization is oriented

along the axis z for $z \ll -L$, and points in the opposite direction for $z \gg L$. In what follows we assume that the DW width L is less than the Fermi wave length λ_F of the charge carriers. This limiting case is appropriate for DWs formed at constrained magnetic contacts, in particular for low-density magnetic semiconductors, where λ_F can be quite large. For the description of the conduction electrons in the semiconductor we assume a parabolic band model. Magnetic polarization of the wire is associated with splitting of the spin-up and spin-down electron bands (we take the quantization axis along z).

Due to the spatial variation of the magnetization $\mathbf{M}(\mathbf{r})$, spin-flip scattering of electrons may occur within the domain wall. In addition, for a sharp DW the spin-up electrons propagating along the axis z are reflected from the effective potential barrier at $z=0$. Hence, the strongest effect of DWs on the electronic transport can be expected in the case of a full spin polarization of the electron gas, i.e., when there are no spin-down electrons at $z < 0$, and no spin-up electrons at $z > 0$. This limit is reached when $JM_0 > E_F$, where J is the exchange integral, and E_F is the Fermi energy in the absence of magnetization. We recall that E_F characterizes the total electron density n of the semiconducting material, $n = (2mE_F)^{3/2} / 3\pi^2 \hbar^3$, where m is the electron effective mass. Hence, the condition ($JM_0 > E_F$) of full spin polarization becomes particularly satisfied when a depletion region near the DW exists.

As mentioned above, the condition of sharp DW means that the wall width is smaller than the electron Fermi wavelength, i.e., $k_F L < 1$, where k_F is the electron Fermi wave vector. This condition can be easily fulfilled in semiconductors, especially in the case of low electron concentration. In addition, when DW is laterally constrained, the number of quantum transport channels can be reduced substantially. In the extreme case only a single conduction channel can be active. The corresponding condition is $k_F L_c < 1$, where L_c is the wire width. This condition can be easily obeyed in semiconductors with low density of carriers.

An important element of the model is the presence of spin-orbit interaction. Under the condition of full spin polarization, the spin-flip scattering provides mixing of different spin channels, that is responsible for the transfer of electrons through the domain wall. Thus, one can expect strong influence of spin-orbit interaction on the total resistance. In the following we assume the spin-orbit interaction in the form of Rashba term. Such an interaction is usually associated with the asymmetric form of the confining potential leading to size quantization in quantum wells and wires. The model Hamiltonian we analyze in this work has the form

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} - JM_z(z)\sigma_z - JM_x(z)\sigma_x + i\alpha\sigma_x \frac{d}{dz}, \quad (1)$$

where α is the parameter of spin-orbit interaction, whereas σ_x and σ_z are the Pauli matrices. We choose the axis x to be normal to the wire and assume that the magnetization in the wall rotates in the x - z plane. The Rashba spin-orbit interaction in Eq. (1) corresponds to the axis y perpendicular to the substrate plane. The magnetization vector rotates then in the

substrate plane. Although the one-dimensional model describes only a single-channel quantum wire, it is sufficient to account qualitatively for some of the recent observations. In addition, the present model can be generalized straightforwardly to the case of a wire with more conduction channels (large width and/or higher carrier concentration).

Our treatment is based on the scattering states. For electrons incident from left to right, the asymptotic form of such states (taken sufficiently far from DW, $|z| \gg L$) is

$$\psi_{kR}(z) = \frac{e^{ikz}}{D_k} \begin{pmatrix} M_k \\ \alpha k \end{pmatrix} + \frac{r e^{-ikz}}{D_k} \begin{pmatrix} M_k \\ -\alpha k \end{pmatrix} + \frac{r_f e^{i\kappa z}}{D_\kappa} \begin{pmatrix} i\alpha\kappa \\ M_\kappa \end{pmatrix}, \quad z \ll -L, \quad (2)$$

$$\psi_{kR}(z) = \frac{t_f e^{ikz}}{D_k} \begin{pmatrix} \alpha k \\ M_k \end{pmatrix} + \frac{t e^{-\kappa z}}{D_\kappa} \begin{pmatrix} M_\kappa \\ -i\alpha\kappa \end{pmatrix}, \quad z \gg L. \quad (3)$$

In Eqs. (2) and (3) k and κ are defined as $k = [2m(E + M)]^{1/2} / \hbar$ and $\kappa = [2m(M - E)]^{1/2} / \hbar$, respectively, whereas the other parameters are $M_k = M + (M^2 + \alpha^2 k^2)^{1/2}$, $M_\kappa = M + (M^2 - \alpha^2 \kappa^2)^{1/2}$, $D_k = (M_k^2 + \alpha^2 k^2)^{1/2}$, and $D_\kappa = (M_\kappa^2 + \alpha^2 \kappa^2)^{1/2}$. Here, M is defined as $M = JM_0$ and E denotes the electron energy.

Due to spin-orbit interaction, electron states are superpositions of spin-up and spin-down components. For simplicity, we call them in the following either spin-up or spin-down waves, because they reduce to such waves in the limit of vanishing spin orbit interaction. Thus, the scattering state (2) and (3) describes the spin-up wave incident from $z = -\infty$ to the right, which is partially reflected and partially transmitted into the spin-up and spin-down channels. The coefficients t and t_f are the transmission amplitudes without and with spin reversal, respectively, whereas r and r_f are the corresponding reflection amplitudes. Even though there are no minority carriers far from the domain wall, the corresponding wavefunction components exist in the vicinity of the domain wall and decay exponentially in the bulk. Similar form applies to the scattering states ψ_{kL} describing electrons incident from the right to the left.

When $kL \ll 1$, the reflection and transmission coefficients can be calculated analytically. Upon integrating the Schrödinger equation $H\psi = E\psi$ [with the Hamiltonian given by Eq. (1)] from $z = -\delta$ to $z = +\delta$, and assuming $L \ll \delta \ll k^{-1}$, one obtains

$$\frac{d\psi_{kj}}{dz} \Big|_{z=+\delta} - \frac{d\psi_{kj}}{dz} \Big|_{z=-\delta} + \frac{2m\lambda}{\hbar} \sigma_x \psi_{kj}(z=0) = 0 \quad (4)$$

for each scattering state ($j=R,L$), where

$$\lambda \approx \frac{J}{\hbar} \int_{-\infty}^{\infty} dz M_x(z). \quad (5)$$

Equation (4) has the form of a spin-dependent condition for electron transmission through a δ -like potential barrier located at $z=0$ and was obtained assuming $kL \ll 1$. The magnitude of the parameter λ defined in Eq. (5) can be estimated as $\lambda \approx JM_0 L / \hbar = ML / \hbar$.

Using the full set of scattering states, together with the wave function continuity condition, one can find a set of

equations for the transmission amplitudes t and t_f . Since the wavefunction component corresponding to conserved electron spin decays exponentially away from the wall, only the spin-flip amplitude t_f determines the electric current in the wire. Let us denote the velocity of the incident electrons by v , $v=k/m$, and by ν the corresponding quantity for the exponentially decaying wave component, $\nu=\kappa/m$. From the Schrödinger equation two equations are deduced for the transmission amplitudes t and t_f , namely

$$\begin{aligned} & \left[ivM_k - \nu M_\kappa - 2i\lambda\alpha\kappa - \frac{\alpha^2\kappa(v+iv)(M_\kappa k - iM_k\kappa)}{i\alpha^2 k\kappa + M_k M_\kappa} \right] \times \frac{t}{D_\kappa} \\ & + \left[2i\alpha v k + 2\lambda M_k + \frac{\alpha\kappa(v-iv)(\alpha^2 k^2 + M_k^2)}{i\alpha^2 k\kappa + M_k M_\kappa} \right] \frac{t_f}{D_k} \\ & = \frac{2ivM_k}{D_k} - \frac{2i\alpha^2 k\kappa(D_k + M_k)}{D_k(i\alpha^2 k\kappa + M_k M_\kappa)}, \end{aligned} \quad (6)$$

$$\begin{aligned} & \left[\frac{i\alpha v k M_\kappa}{M_k} + \frac{\alpha\nu(\alpha^2 k^2 + M_k M_\kappa)(M_\kappa k - iM_k\kappa)}{M_k(i\alpha^2 k\kappa + M_k M_\kappa)} - i\alpha\nu\kappa \right. \\ & \left. - 2\lambda M_\kappa \right] \frac{t}{D_\kappa} + \left[\frac{iv\alpha^2 k^2}{M_k} - ivM_k - 2\alpha\lambda k \right. \\ & \left. + \frac{\nu(\alpha^2 k^2 + M_k^2)^2}{M_k(i\alpha^2 k\kappa + M_k M_\kappa)} \right] \frac{t_f}{D_k} = \frac{2\alpha k\nu(\alpha^2 k^2 + M_k M_\kappa)}{D_k(i\alpha^2 k\kappa + M_k M_\kappa)}. \end{aligned} \quad (7)$$

In the absence of spin-orbit interaction, $\alpha=0$, one finds

$$t = \frac{2v(v+iv)}{(v+iv)^2 + 4\lambda^2}, \quad t_f = \frac{4i\lambda v}{(v+iv)^2 + 4\lambda^2}. \quad (8)$$

In the limit of $\nu \gg v$ and $\lambda \ll \nu$ (low density of carriers and small spin-orbit interaction) another limiting formula is derived

$$t_f = -\frac{4iv\lambda^2}{\nu^2(\lambda - i\alpha\nu\kappa/M)}. \quad (9)$$

In general, the coefficient t_f can be found analytically but the corresponding formula is rather cumbersome.

In the limit of $\lambda \rightarrow 0$ (very thin DW), the transmission through the wall vanishes, which corresponds to the complete reflection of electrons from the wall. Thus, at first glance one might expect that a nonzero spin-orbit interaction mixes the spin channels and leads to nonvanishing transmission through the wall, even in the limit of very thin domain wall. This is however not the case since the matching condition for the wave functions at $z < L$ and $z > L$ requires that both incident and transmitted waves are certain superpositions of spin-up and spin-down components. On the other hand, Eq. (9) indicates that transmission through the wall decreases with increasing strength of the spin-orbit interaction.

III. RESISTANCE OF THE DOMAIN WALL

To calculate the conductance of the system, we use the Büttiker-Landauer formula, which can be simplified substan-

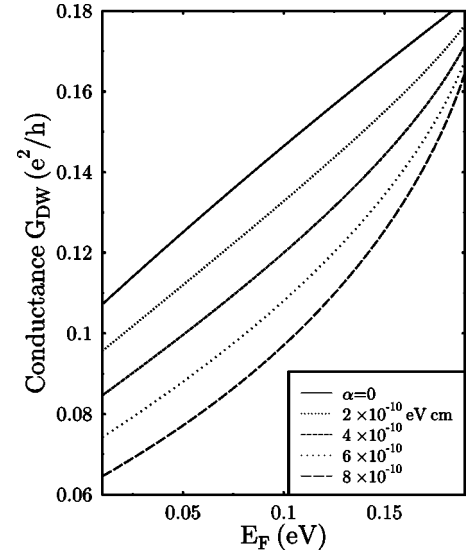


FIG. 1. Conductance of a magnetic wire with a single domain wall vs Fermi energy of electrons. Different curves correspond to different values of the spin-orbit coupling parameter α .

tially due to the suppression of all channels, but spin-flip through the wall. (The derivation of such a formula for transmission through the wall in the case of all nonvanishing channels has been done in Ref. 23.) Thus one obtains

$$G = \frac{e^2}{2\pi\hbar} |t_f|^2. \quad (10)$$

Due to the asymptotic current conservation, the conductivity is determined by the propagating (nondecaying) component of the transmitted wave. Using Eq. (8) one finds for vanishing spin-orbit interaction

$$G = \frac{8e^2}{\pi\hbar} \frac{\lambda^2 v^2}{(v^2 - \nu^2 + 4\lambda^2)^2 + 4\nu^2 \nu^2}. \quad (11)$$

Here, all the velocities are taken at the Fermi level.

Figure 1 shows the calculated dependence of the electrical conductance on the Fermi-energy E_F in the general case. The calculations were performed assuming the following values of the relevant parameters: $m=0.6m_0$ (where m_0 is the free electron mass), $JM_0=0.2$ eV, and $L=10^{-8}$ cm. These parameters correspond to GaMnAs semiconductor, and satisfy the condition $JM_0 > E_F$ for $E_F < 0.2$ eV.

We can estimate the magnitude of parameter α by taking the value of the spin-orbit (SO) splitting $\Delta E_{SO} \approx ak$, where the momentum k is related to the density of carriers N_s as $k=(2\pi N_s)^{1/2}$. Assuming $\Delta E_{SO}=0.5$ meV for $N_s=10^{11}$ cm $^{-2}$ as a characteristic value for GaAs-GaAlAs heterostructures,²⁶ one obtains $\alpha \approx 6.3 \times 10^{-10}$ eV cm.

From Fig. 1 it is clear that the conductance increases monotonically with increasing E_F because the barrier is felt smaller by electrons having higher energy. Furthermore, the conductance of a magnetic wire with DW diminishes with increasing strength of the spin-orbit interaction.

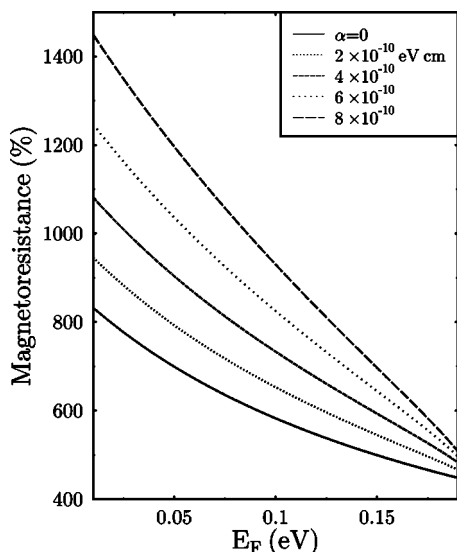


FIG. 2. Magnetoresistance of the wire with a domain wall vs Fermi energy for different values of α .

The dependence of the magnetoresistance on the Fermi energy E_F is presented in Fig. 2 for different values of the parameter α . The magnetoresistance is calculated with respect to the state without DW, $MR = R_{DW}/R_0 - 1$, where R_{DW} is the resistance of the wire with DW and $R_0 = 2\pi\hbar/e^2$ is its resistance in the absence of the wall (only spin-up channel is active). For our choice of parameters, the magnetoresistance is rather high and increases substantially with spin-orbit interaction.

The magnetoresistance measurements on magnetic semiconductors are usually performed at low temperatures because the corresponding Curie temperature is rather low. At such conditions, one can expect a significant contribution of the localization corrections to the conductivity. The role of the localization in the case of smooth DWs (for $kL \gg 1$) has been studied before,^{27,28} and it was shown that the localization corrections are suppressed by an effective gauge field of the wall. This means that the contribution of the wall to resistance is negative, and the corresponding magnetoresistance is positive.

We have analyzed the role of localization corrections in the case of sharp DW. Qualitatively, it can be described as the DW induced suppression of the quantum interference in triplet Cooperon channel.²⁹ The singlet channel in ferromagnets is strongly suppressed by the internal magnetization.³⁰ The suppression of the interference by DWs is related to dephasing of the wave function of electron transmitted through the barrier.^{31,32} If the transmission through the wall is small, the corresponding dephasing length roughly equals to the distance of electron moving from a point z (within the constriction) to the domain wall position ($z=0$), and the dephasing time is $\tau_{dw}(z) \sim z^2/D$, where D is the diffusion coefficient. After averaging over z of the localization correction $\delta G(z)$,

we find that the characteristic dephasing length L_0 is the constriction length itself, $\delta G_{dw} \approx -e^2 L_0 / \pi\hbar$. In the case of sharp DWs, the localization correction diminishes the magnetoresistance due to the reflection from the wall, since it has a different sign.

IV. CONCLUSIONS

We have presented a theoretical description of the resistance of a semiconducting magnetic nanojunction with a constrained DW in the case of a full spin polarization of electron gas. In the limit of $kL \ll 1$, the electron transport across the wall was treated effectively as electron tunneling through a spin-dependent potential barrier. For such a narrow and constrained DW, the electron spin does not follow adiabatically the magnetization direction, but its orientation is rather fixed. However, DW produces some mixing of the spin channels. The spin-orbit interaction essentially enhances the magnetoresistance, whereas the localization corrections play the opposite role. However, the localization corrections can be totally suppressed by the spin-orbit interaction.³⁰ This indicates that the spin-orbit interaction can play an important role and can lead to large enhancement of the magnetoresistance effect.

In our calculations we assumed the one-band model with parabolic energy spectrum and with the spin-orbit interaction in the form of Rashba Hamiltonian. Such a model can describe n -type semiconductor films on a substrate or asymmetric semiconducting quantum wells. Some real III-V magnetic semiconductors like GaMnAs are known to be of p -type and to have rather complex band structure consisting of several hole subbands with the spin-orbit interaction included in the hole Hamiltonian. The parabolic approximation is still valid for the hole energy $E \ll \Delta_{SO}$, where Δ_{SO} is the energy of SO splitting. However, in a general case, the theory of hole tunneling through a domain wall in p -GaMnAs needs special consideration.

In the qualitative discussion of the localization effects, we assumed that DWs suppress the localization corrections via the gauge field acting on electrons within DWs. This assumption corresponds to 3D or effectively 2D and 1D cases (the decoherence length is smaller than the width of the wire). We can expect a significantly reduced effect of DWs on the localization corrections in a one-channel ballistic wire. However, any deviation from one-dimensionality combined with nonideality of the magnetic profile in DWs leads to an increased effect of DWs on the localization corrections, whereas the SO interaction suppresses the triplet Cooperon correction as discussed in Sec. III.

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