Anisotropic spin and charge transport in presence of spin-orbit interaction

T. P. Pareek
Max-Planck-Institute für Mikrostrukturphysik, Weinberg 2, D-06120 Halle, Germany
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We explore spin and charge transport phenomena in a two-dimensional electron gas (2DEG) in presence of spin-orbit coupling connected to two ideal Ferromagnetic leads with parallel magnetization. It is shown that the spin polarization transported through the 2DEG depends on the absolute direction of magnetization in a coordinate system defined by plane of 2DEG and normal to it. Conductance is also shown to be anisotropic.

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The growing field of spintronics has attracted a lot of interest after the proposal of the spin-field effect transistor (spin-FET) by Datta and Das.[1] The Datta-Das spin-FET is a hybrid structure of type FM1-2DEG-FM2, where 2DEG is a two-dimensional electron gas of a narrow gap semiconductor (InAS) and FM1 and FM2 are injector and detector ferromagnetic contacts. The working of this device relies on the manipulation of electronic spin state in 2DEG with the electric field of an external gate electrode. Essential for this mechanism is field dependent spin-orbit coupling, which is relatively large and well established.[2] It is now generally accepted that the spin-orbit coupling in narrow-gap 2DEG is governed by the Rashba Hamiltonian.[3] For a 2DEG lying in xy plane (see Fig. 1) the Rashba spin-orbit interaction has the form $H_R=\alpha(\mathbf{k}\times\boldsymbol{\sigma})\hat{z}$, with $\mathbf{k}$ being the momentum vector, $\boldsymbol{\sigma}$ Pauli matrices, and $\hat{z}$ is a unit vector perpendicular to 2DEG plane. The Rashba spin-orbit causes spin splitting for $\mathbf{k}\neq 0$, $\Delta E=2\alpha k$, which is linear in momentum. The Rashba splitting is due to absence of space inversion symmetry. However the exchange splitting in ferromagnets is due to the breaking of time reversal symmetry. Therefore it is natural to expect that spin and charge transport properties of a hybrid structure like spin-FET, which combines elements with different symmetry properties, may be different than the standard mesoscopic structures consisting of elements with same symmetry, for, e.g., all metal mesoscopic structures.

Motivated by this, in this paper we study the spin and charge transport of a FM1-2DEG-FM2 system sketched in Fig. 1. A natural reference frame for the Fig. 1 is defined by the plane of 2DEG (we call it the xy plane) and the normal to this plane , i.e., the z axis. The polarization of the ferromagnets FM1 and FM2 are equal in magnitude and parallel to each other but points in a direction (θ,ϕ), i.e., $\mathbf{P}_1=\mathbf{P}_2=P_0$ $(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)$, with θ and ϕ being the usual spherical angles. The question addressed here is the following: does the spin polarization transported through 2DEG from FM1 to FM2, and the charge transport, i.e., conductance depend on (θ,ϕ)? We show through a combination of analytical and numerical calculation, that transported spin polarization and charge conductance are anisotropic, i.e., depends on angle θ and ϕ. These anisotropies are present irrespective of the Hamiltonian considered being an effective mass Hamiltonian or tight binding Hamiltonian.[4–6]

The Hamiltonian of a 2DEG in presence of Rashba spin-orbit coupling reads[3]

$$H=-\frac{\hbar\nabla^2}{2m}+\alpha(\boldsymbol{\sigma}\times\mathbf{k})\cdot\hat{\mathbf{z}}, \tag{1}$$

where α is the Rashba spin-orbit interaction parameter. We write the above Hamiltonian in the matrix form which is more convenient for the study of spin transport

$$H=\frac{1}{2}(B_0\mathbf{I}+\mathbf{B}_R\cdot\boldsymbol{\sigma}), \tag{2}$$

where $\mathbf{I}$ is the 2×2 identity matrix, $B_0=\hbar^2(k_x^2+k_y^2)/m$ and the vector is $\mathbf{B}_R=2\alpha(k_y\hat{\mathbf{x}}-k_x\hat{\mathbf{y}})$. Note that the magnitude and direction of $\mathbf{B}_R$ is determined by wave vector $\mathbf{k}$. Infact the direction of $\mathbf{B}_R$ is always perpendicular to the instantaneous wave vector $\mathbf{k}$.

An appropriate physical quantity to study the spin transport is the polarization vector $\mathbf{P}=\langle\boldsymbol{\sigma}\rangle$, where angular bracket represents the ensemble averaging. With this definition one can immediately write down the equation of motion for polarization vector

$$\frac{d\mathbf{P}}{dt}=\frac{d\langle\boldsymbol{\sigma}(t)\rangle}{dt}=-\frac{i}{\hbar}\langle\boldsymbol{\sigma}(t)H-H\boldsymbol{\sigma}(t)\rangle. \tag{3}$$

Simplifying the above equation using Eq. (2) one gets

$$\hbar\frac{d\mathbf{P}}{dt}=\mathbf{B}_R\times\mathbf{P}. \tag{4}$$

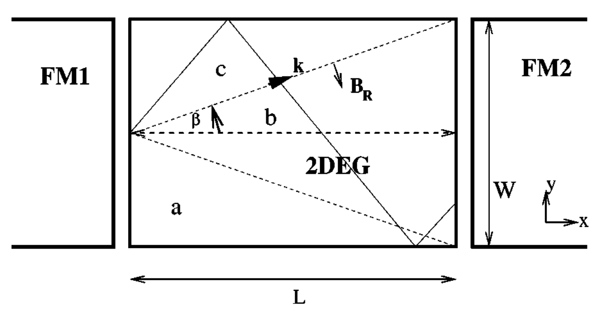

FIG. 1. A 2DEG connected to two ideal Ferromagnetic leads. The three region shown are respectively, (a), (b), and (c). Trajectories lying in region (b) (depicted as dashed lines) reaches the FM2 ballistically. Trajectories lying in regions (a) and (c) (shown as solid lines) scatters from boundary before reaching FM2.

Equation (4) is well known in the literature and is fully quantum mechanical and holds even if $\mathbf{B}_R$ is time dependent.[7] Equation (4) can be solved analytically when the field $\mathbf{B}_R$ is a constant vector, the most general solution is given as

$$\mathbf{P}(t)=\mathbf{P}_0\cos(\omega_R t)+2\hat{\boldsymbol{B}}_R(\hat{\boldsymbol{B}}_R\cdot\mathbf{P}_0)\sin^2(\omega_R t/2)+(\hat{\boldsymbol{B}}_R\times\mathbf{P}_0)\sin(\omega_R t), \tag{5}$$

where $\mathbf{P}_0$ is the initial polarization imposed by ferromagnet FM1 and $\omega_R=B_R/\hbar$ is precession frequency (precession angle $\gamma=\omega_R t$). Since we are interested in the transport properties when the polarization vector of injector and detector ferromagnets (FM1 and FM2 in Fig. 1) are equal and parallel, hence by projecting $\mathbf{P}(t)$ on $\mathbf{P}_0$ we obtain the required solution

$$\mathbf{P}(t)\cdot\mathbf{P}_0=|\mathbf{P}_0|^2\cos(\omega_R t)+2(\mathbf{P}_0\cdot\hat{\boldsymbol{B}}_R)^2\sin^2(\omega_R t/2), \tag{6}$$

where $\omega_R=B_R/\hbar\equiv 2\alpha k_f/\hbar$. For a given injection angle β as shown in Fig. 1, Eq. (6) simplifies to

$$\mathrm{pol}(\theta,\phi,\omega_R t,\beta)\equiv\frac{\mathbf{P}(t)\cdot\mathbf{P}_0}{|\mathbf{P}_0|^2}=\cos(\omega_R t)+2\sin(\beta-\phi)^2\sin(\theta)^2\sin^2(\omega_R t/2). \tag{7}$$

The quantity $\mathrm{pol}(\theta,\phi,\omega_R t,\beta)$ is a measure of spin polarization (for a given injection angle β) transferred from FM1 to FM2 through the 2DEG. Equation (7) is a general solution for any given θ and ϕ, for particular θ and ϕ solution can be found in standard text.[7] In Eq. (7) t is the time electron takes to reach the output terminal. Since the electron are injected over the range $-\pi/2\leqslant\beta\leqslant\pi/2$, we need to make an average over all possible values of injection angle β. To do this we proceed as follows: We notice that depending upon injection angle β electron reaches the boundary ballistically (dashed trajectory in Fig. 1) or with scattering (solid trajectory in Fig. 1) from the boundaries. Hence we need to calculate t accordingly for different values of β. Therefore we divide the integration over β in three regimes, namely, (a) $-\pi/2\leqslant\beta\leqslant-\tan^{-1}(W/L)$, (b) $-\tan^{-1}(W/L)\leqslant\beta\leqslant-\tan^{-1}(W/L)$, and (c) $\tan^{-1}(W/L)\leqslant\beta\leqslant\pi/2$. The regimes (a) and (c) correspond to the trajectories which suffers scattering from boundary while trajectories in regime (b) propagates ballistically. Since trajectories lying in regime (b) propagates ballistically therefore the time to reach the output terminal is $t=L/\cos(\beta)$ (see Fig. 1 dashed line). For trajectories lying in regimes (a) and (c) the electron scatters from the boundary at least once before reaching the out put terminal (FM2), hence for these values of β, we assume that the electrons diffuse along the channel with a mean free path $W/\sin(\beta)$ (later in our exact numerical simulation we will see that this approximation is quite reasonable). Hence the time to reach the boundary is given as $t=[2L^2\sin(\beta)]/(v_f W)$. Using the corresponding value of t for regimes (a), (b), and (c),we obtain precession angle $\gamma=\omega_R t$,

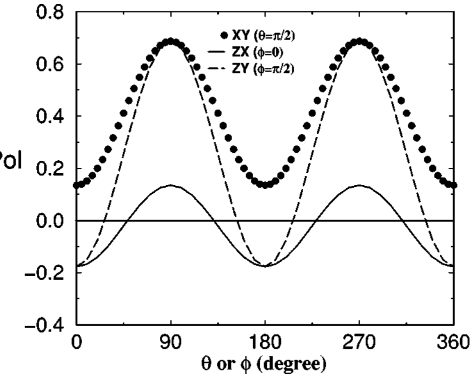

FIG. 2. Polarization transported from FM1 to FM2 through 2DEG as a function of angle, calculated using Eqs. (7) and (8) as explained in text. Where $\tilde{L}=\tilde{W}=50/2\pi,\tilde{\alpha}=0.06$.

$$\omega_R t=\begin{cases}\dfrac{2\alpha k_f L}{v_f\cos(\beta)}\equiv\dfrac{2\pi\tilde{\alpha}\tilde{L}}{\cos(\beta)} & \beta\in\left\{-\tan^{-1}\left(\dfrac{W}{L}\right),\tan^{-1}\left(\dfrac{W}{L}\right)\right\},\\[2ex] \dfrac{2\alpha k_f L^2\sin(\beta)}{v_f W}\equiv\dfrac{4\pi\tilde{\alpha}\widetilde{L^2}\sin(\beta)}{\tilde{W}} & \beta\in\left\{\pm\dfrac{\pi}{2},\pm\tan^{-1}\left(\dfrac{W}{L}\right)\right\},\end{cases} \tag{8}$$

where $\tilde{\alpha}=\alpha k_f/E_f$ is dimensionless Rashba parameter (E_f is Fermi energy) and $\tilde{L}=L/\lambda_f$ and $\tilde{W}=W/\lambda_f$ are the length and width of the channel in units of Fermi wavelength. Substituting these values of $\omega_R t$ in Eq. (7) and performing the integration over β, we obtain polarization as function of $\theta,\phi,\tilde{\alpha}$ for a given $\tilde{L}$ and $\tilde{W}$. Equation (7) together with Eq. (8) can be used to calculate the transported polarization for any given direction (θ,ϕ), however, for clarity and simplicity we present results for three specific cases corresponding to different values of θ and ϕ, namely, (i) $\theta=\pi/2$, ϕ is variable, i.e., polarization of FM1 and FM2 is rotated in the xy plane (the plane formed by 2DEG) (ii) $\phi=0$, θ is the variable corresponding to the rotation in the xz plane, (iii) $\phi=\pi/2$, θ is the variable corresponding to the rotation in yz plane. For these three different cases the transported polarization given by Eq. (7) is shown in Fig. 2 as a function of angle. It is clearly seen from Fig. 2 that transported polarization is anisotropic.

The amplitude of oscillation tells us about the spin coher-

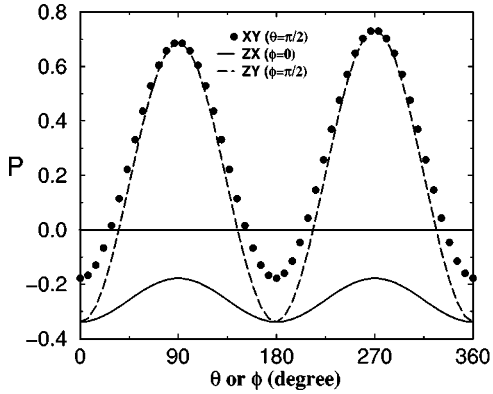

FIG. 3. Results of numerical simulation for polarization for ballistic system. The numerical simulation were performed on a 50×50 lattice within tight binding model. The tight binding Rashba parameter is given by $\lambda_{so}=\tilde{\alpha} t k_f a/2=0.03$, FM exchange splitting is $\Delta/E_f=0.5$, and $k_f a=1$. These parameters were chosen in such a way that they correspond to the parameters of Fig. 1, as explained in text (analytical result).

ence and since this is different for all the three cases, it signifies that the spin coherence is also affected anisotropically. References 6 discusses the spin coherence(which is related with the amplitude of oscillation) when the injected current is unpolarized since the contacts were nonmagnetic hence the question of transport of spin polarization does not arise. In fact, it is seen from Fig. 2 that amplitude of oscillation is larger for cases (i) and (iii), compared to case (ii). The absolute magnitude of oscillation is always smaller than one implying even in ballistic transport spin dephasing takes place due to the boundary scattering. Though in our analytical calculation boundary scattering was treated as diffusive, however, we will see in the exact numerical calculation that a perfectly reflecting boundary also leads to dephasing.

To further strengthen our results we performed numerical simulation on a tight binding square lattice of lattice spacing a with N_x sites along x axis and N_y sites along the y axis. For the tight binding Hamiltonian the Rashba spin-orbit coupling is given by $\lambda_{so}=\alpha/2a=\tilde{\alpha} t k_f a/2$ (see Refs. 6,8). We fix $t=1$ (hopping) and $k_f a=1$ (ballistic case) for the numerical simulation in the tight binding model. Once t and $k_f a$ are fixed the other parameters for the tight binding model which would correspond to the parameters of Fig. 1 are given as $N_x=2\pi\tilde{L}=50$, $N_y=2\pi\tilde{W}=50$, and $\lambda_{so}=\tilde{\alpha} t k_f a/2=0.03$. With these set of parameters we calculate spin resolved conductance for a given polarization direction (θ,ϕ) of ferromagnets, within Landauer-Büttiker formalism.[5,6,8,10] Using the spin resolved conductance we define the polarization as

$$P=\frac{G_{\mathrm{sc}}-G_{\mathrm{sf}}}{G_{\mathrm{sc}}+G_{\mathrm{sf}}}, \tag{9}$$

where G_{sc} and G_{sf} are spin-conserved and spin-flip conductance, respectively. The quantity P in Eq. (9) corresponds to the quantity given in Eq. (7) and also lies between $+1$ and -1. This is plotted in Fig. 3.

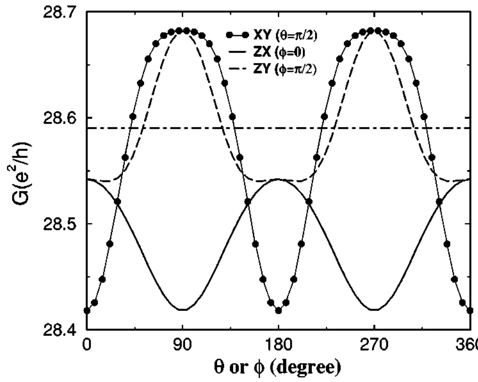

FIG. 4. Conductance as a function of angle corresponding to the Fig. 3. The dot-dashed line is the conductance in absence of spin-orbit interaction.

We see that the agreement between Fig. 2, i.e., analytical calculation, and Fig. 3 (simulation) is quite good. The slight quantitative mismatch is due to the fact that numerical simulation was done for hard wall confining potential in y direction which leads to specular reflection, while in analytical calculation scattering from the boundary was treated as diffusive. Therefore it is clear that the anisotropy in spin transport is present in the continuum model (effective mass Hamiltonian) as well as in tight binding model and is not an effect of reduced symmetry of tight binding model.[6] Recently anisotropy in polarization transport have been observed for holes injected into a quantum well.[9] However, the mechanism is not clear, see Ref. 9. Our results suggest that for electrons, spin-orbit interaction can lead to anisotropy in polarization transport but we cannot make definite statement regarding the experimental result Ref. 9 since the effect there is related to holes.

Now since conductance of FM/2DEG/FM depends on the polarization of electrons reaching the output terminal, hence it is expected that conductance should also be anisotropic. This is clearly visible in Fig. 4, where we have plotted the total conductance, i.e., $G=G_{\mathrm{sc}}+G_{\mathrm{sf}}$ corresponding to Fig. 3, as function of polarization angle. It should be noted that the conductance is symmetric with respect to angle θ or ϕ which is consistent with Büttiker symmetry relation for charge transport.[10] It is instructive to note that the conductance does not depend on polar angle θ or ϕ in absence of spin-orbit interaction as is seen from the Fig. 4 (dot-dashed straight line). This clearly shows that the anisotropies are a consequence of rotational symmetry breaking by spin-orbit interaction. In a recent paper Matsuyama *et al.*[11] studied conductance oscillation in similar system arising due to Fabry-Perot resonances. In particular they showed that conductance oscillates as a function of carrier density for a fixed magnetization direction, i.e., either parallel to the x axis or y axis [see Figs. 11(a) and 11(b) in Ref. 11]. The oscillation reported in this work arises due to a change in magnetization direction while keeping all other parameters fixed, see Fig. 4, where the conductance is shown as a function of θ and ϕ. Also we would like to point out that Ref. 12 reports experimental results for change in resistance when the magnetization of FM1 and FM2 is changed from parallel to antiparallel con-

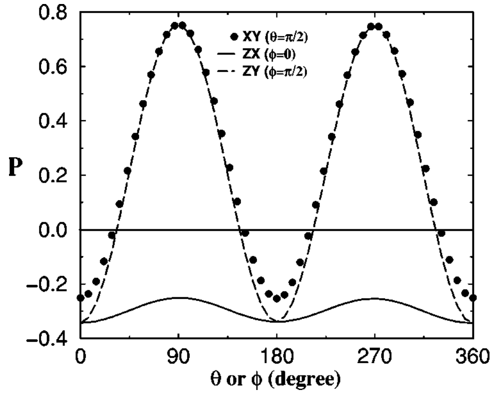

FIG. 5. Polarization as a function of angle for diffusive case. Here $k_f l = 10$, where l is the mean free path. Configuration averaging was performed over 15 different configuration. The other parameters are same as in Fig. 3.

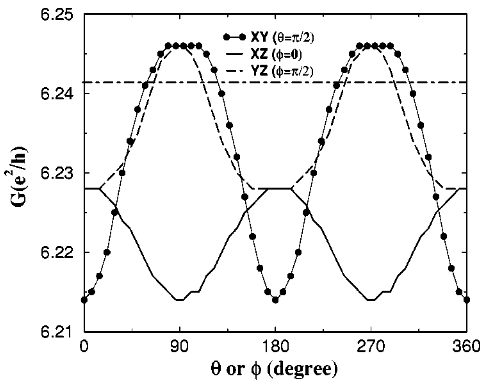

FIG. 6. Conductance as function of angle corresponding to the Fig. 5. Here $k_f l = 10$, where l is mean free path.Configuration averaging was performed over 15 different configuration. The other parameters are the same as in Fig. 3. The dot-dashed straight line shows the conductance in the absence of spin-orbit interaction.

figuration. Our results for conductance calculation pertains to the situation when the magnetization of FM1 and FM2 are always parallel but points in a direction (θ,ϕ) as explained in the introduction. Hence our result is not related with the experimental data of Ref. 12.

The results presented above were in the ballistic regime. To verify that these results survives in a diffusive case we show polarization and conductance in Figs. 5 and 6, respectively, for the diffusive case. We have taken Anderson model for disorder with width $3|t|$, corresponding to a mean free path of $l = 10a$. The other parameters are same as those for Figs. 3 and 4. It is clearly seen that the anisotropy survives even in the diffusive case.

It is instructive to compare Fig. 3 for ballistic transport and Fig. 5 for diffusive transport. It is seen that the polarization which is transported is not affected much by the presence of disorder which is consistent with the fact that the Rashba spin-orbit interaction is independent of disorder strength.

In summary we have demonstrated that spin and charge transport in the presence of Rashba spin-orbit interaction are anisotropic. These anisotropies are consequence of breaking of rotational invariance due to the presence of the spin-orbit interaction.

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[1] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
[2] G. Lommer *et al.*, Phys. Rev. Lett. **60**, 728 (1988); B. Das *et al.*, Phys. Rev. B **41**, 8278 (1990); J. Nitta *et al.*, Phys. Rev. Lett. **78**, 1335 (1997).
[3] Yu.A. Bychkov and E.I. Rashba, JETP Lett. **39**, 78 (1984).
[4] L.W. Molenkamp, G. Schmidt, and G.E.W. Bauer, Phys. Rev. B **64**, 121202 (2001).
[5] T.P. Pareek and P. Bruno, Pramana, J. Phys. **58**, 293 (2002).
[6] T.P. Pareek and P. Bruno, Phys. Rev. B **63**, 165424 (2001); T.P. Pareek and P. Bruno, *ibid.* **65**, 241305 (2002).
[7] C.P. Slichter, *Principles of Magnetic Resonance* (Harper and Row, New York, 1963), p. 19.
[8] F. Mireles and G. Kirczenow, Phys. Rev. B **64**, 024426 (2001).
[9] D.K. Young *et al.*, Appl. Phys. Lett. **80**, 1598 (2002).
[10] M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986); IBM J. Res. Dev. **32**, 317 (1988).
[11] T. Matsuyama *et al.*, Phys. Rev. B **65**, 155322 (2002).
[12] C.M. Hu *et al.*, Phys. Rev. B **63**, 125333 (2001).