Recent progress in chirality studies of non-relativistic, few interacting particle systems

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This work reviews recent achievements in the understanding of highly excited, interacting few-particle systems with a well defined internal helicity. To contrast theory with experiments we consider specifically the spectrum of two interacting continuum electrons with a positive (or a negative) chirality that move in the field of a positive ion. This doubly excited state is achieved upon the absorption of a circular photon with the appropriate helicity by an isotropic target. The dependence of the two-electron spectrum on the sign of the helicity of the exciting photon is utilized to study internal phase relations and mirror-reflection symmetries of the two-electron wave function.

1 Introduction

The understanding of correlated electronic systems is of a fundamental interest for a number of branches of physics [1]. In particular, the description of correlated systems poses a challenging and a fascinating task for theoretical physics: While the presence of electronic correlation induces a number of important phenomena (such as the metal-insulator transition) it precludes on the other hand an exact treatment, even for a system of three coupled particles. In view of this, symmetry considerations on general ground attain a special importance for correlation studies. In this report we consider a simple, non-relativistic quantum mechanical system consisting of three interacting particles above the total fragmentation threshold. From the structure of the Schrödinger equation we derive symmetry properties that are generally valid and discuss the interrelation between the spin and the spatial part of the wave function as introduced by the particle-exchange symmetry requirement. In addition, we point out a connection, akin to *many* body system, between the internal phase relations of the interacting three-body wave function and the mirror reflection symmetry with respect to a given plane. The proposed ideas and relations can be assessed experimentally by studying the spectrum of an excited three-body system which is created upon the absorption of a one circular photon by a completely isotropic many-electron atom. In this case the helicity of the photon is transferred to the three-body system. The measurable continuum spectrum of this excited helical system is strongly dependent on whether the helicity (of the absorbed photon) is positive or negative. This theoretical prediction has been confirmed by recent experiments [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The purpose of the present report is to show how the helicity dependence of these spectra can be utilized to investigate the internal phase relations within the many-body wave function and to study the mirror-reflection symmetry.

2 General considerations

Throughout this work we consider a non-relativistic quantum system and neglect spin-dependent interactions in the Hamiltonian. As a consequence the spin and the spatial degrees of freedom are decoupled. However, the Pauli principle induces, via symmetry restriction of the spatial part, an influence of the spin state of the system on its spatial evolution.

For the sake of clarity the consideration made in this paper are limited to the case of three interacting particle, e. g. two electrons moving in the field of a positively charged residual ion. In the center of mass system the motion of the two electrons is described by the wave function $\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$. The structure of the time-independent Schrödinger equation dictates that

$$\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \psi_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_2,\boldsymbol{r}_1), \tag{1}$$

$$\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \psi_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(-\boldsymbol{r}_1,-\boldsymbol{r}_2). \tag{2}$$

Furthermore, in absence of external fields the isotropy of space implies

$$\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \psi_{\boldsymbol{k}'_2,\boldsymbol{k}'_1}(\boldsymbol{r}'_2,\boldsymbol{r}'_1) \tag{3}$$

where $\boldsymbol{a}' = \boldsymbol{R}(\alpha,\hat{\boldsymbol{\omega}})\,\boldsymbol{a}$, $\boldsymbol{a} \in \{\boldsymbol{k}_{1/2},\boldsymbol{r}_{1/2}\}$ and $\boldsymbol{R}(\alpha,\hat{\boldsymbol{\omega}})$ is an arbitrary rotation operation defined by the axis $\hat{\boldsymbol{\omega}}$ and the rotation angle α. Eq.(2) states that the wave function is invariant under a grand inversion at the origin. This feature combined with the rotational invariance (Eq.(3)) yields that the wave function is as well invariant under a mirror reflection of all the coordinates *and* the momenta with respect to an arbitrary plane.

Eqs.(1-3) are generally valid. On the other hand, it should be noticed that in general

$$\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) \neq \psi_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_1,\boldsymbol{r}_2), \tag{4}$$

$$\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) \neq \psi_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2). \tag{5}$$

This means that the (spatial) wave function, as derived from the Schrödinger equation is not invariant under an exchange or a mirror reflection of the quantum numbers (the momenta). Therefore, one defines the following expressions:

$$\Psi^{\pm}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{\sqrt{2}}\left[\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) \pm \psi_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_1,\boldsymbol{r}_2)\right], \tag{6}$$

$$\Psi^{\pm,e}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{\sqrt{2}}\left[\Psi^{\pm}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) + \Psi^{\pm}_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)\right], \tag{7}$$

$$\Psi^{\pm,o}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{\sqrt{2}}\left[\Psi^{\pm}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) - \Psi^{\pm}_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)\right]. \tag{8}$$

In addition to the properties (1-3) the wave functions $\Psi^{\pm,e/o}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ possess further symmetries that serve for a classification as symmetric ($\Psi^{+,e/o}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = +\Psi^{+,e/o}_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_1,\boldsymbol{r}_2)$) or antisymmetric functions ($\Psi^{+,e/o}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\Psi^{+,e/o}_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_1,\boldsymbol{r}_2)$), as well as even ($\Psi^{\pm,e}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = +\Psi^{\pm,e}_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$) or odd functions ($\Psi^{\pm,o}_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2) = -\Psi^{\pm,o}_{-\boldsymbol{k}_1,-\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$).

3 Few-particle wave-function symmetries studied by helical photons

As stated above, till now we decoupled spins from spatial degrees of freedom and discussed the symmetry properties of the spatial part only. The total wave function is then a direct product of the spatial part ($\Psi^{\pm,e/o}$) and a two-electron spin part $\chi(1,2)$. The total wave function has to be antisymmetric with respect to exchange of the two electrons. Therefore, if $\chi(1,2)$ is symmetric (triplet state with a unity total spin) the spatial part has to be antisymmetric, whereas if $\chi(1,2)$ is symmetric (singlet state with a vanishing total spin), the spatial part is symmetric. By performing spin-polarized scattering experiments in the singlet and the triplet channel [18, 19, 20, 21, 22] one can deduce on the dynamical phase relations between the functions $\psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\psi_{\boldsymbol{k}_2,\boldsymbol{k}_1}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ (cf. Eq.(4)). Due to space limitation and hence this kind of studies is well documented by now [18, 19, 20, 21, 22, 23, 24, 25], we focus our attention on the analogous problem of finding out phase relations between $\Psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and $\Psi_{\boldsymbol{k}'_1,\boldsymbol{k}'_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ where $\boldsymbol{k}'_1$ and $\boldsymbol{k}'_2$ are the mirror images with respect to a given plane of $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, respectively. As remarked above, in absence of external fields (i.e. if the space is isotropic), this operation combined with an appropriate rotation in (momentum) space corresponds to a reflection at the origin (cf. Eq.(5) and Fig.1).

To connect to existing experiments let us consider the photo double emission (PDE) process. In this reaction an incoming photon with a wave vector $\boldsymbol{k}$, a frequency ω and a polarization vector $\hat{\boldsymbol{\epsilon}}$ ionizes two electrons from an randomly oriented atom. The two electrons escape into the double continuum with wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$. For our purposes the initial bound state of the atom Φ should be completely isotropic for reasons which will become clear below. In first order perturbation theory for the electromagnetic field the PDE cross section $W(\hat{\boldsymbol{\epsilon}},\boldsymbol{k}_1,\boldsymbol{k}_2)$ is obtained from the optical transition amplitude as $W(\hat{\boldsymbol{\epsilon}},\boldsymbol{k}_1,\boldsymbol{k}_2) = C|T(\hat{\boldsymbol{\epsilon}},\boldsymbol{k}_1,\boldsymbol{k}_2)|^2$ where

$$T(\hat{\boldsymbol{\epsilon}},\boldsymbol{k}_1,\boldsymbol{k}_2) = \hat{\boldsymbol{\epsilon}}\cdot\langle\Psi_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{r}_1,\boldsymbol{r}_2)|\boldsymbol{D}|\Phi\rangle, \tag{9}$$

and C is a constant pre-factor [17]. In Eq.(9) $\boldsymbol{D}$ is the dipole operator[1]). This approximation for T is well justified in the VUV regime for the photons.

Let us consider the geometry depicted in Fig.1: The photon is left-hand, circularly polarized with a wave vector $\boldsymbol{k}$. The two electrons recede from the residual ion

[1]) For brevity we omitted the superscripts $\pm,(e/o)$ of Ψ. The appropriate symmetry of Ψ is determined by the nature of Φ, e.g. if Φ is an even, symmetric state Ψ must have an odd parity and should be symmetric with respect to an exchange of the two electrons

with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ that lay in the $x-y$ plane. Without loss of generality we can choose $\hat{\boldsymbol{k}}$ as the z direction and the x direction to be defined by $\hat{\boldsymbol{k}}_1$ (cf. Fig.1).

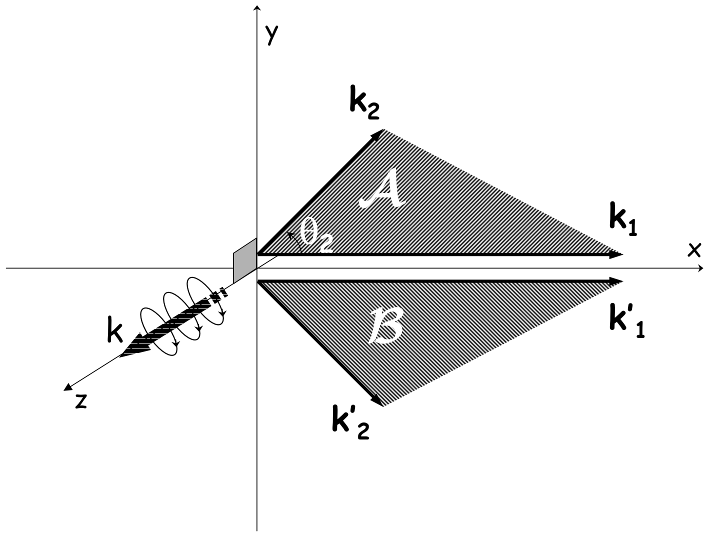

Fig. 1. A schematic representation of the experimental arrangement for a one-photon two-electron continuum transition. The photon is circularly polarized and propagates along the z direction. The two electrons absorb the photon and move in the $x-y$ plane with momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ that span the triangle $\mathcal{A}$. The triangle $\mathcal{B}$ that quantifies the state with wave vectors $\boldsymbol{k}_1'$ and $\boldsymbol{k}_2'$ is the mirror image at the $x-z$ plane of $\mathcal{A}$.

The quantum mechanical state of the two electrons is quantified by a triangle spanned by the wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ (cf. Fig.1). The question which we would like to address is whether we can sense any difference between the states described by the triangle $\mathcal{A}$ and its ($x-z$ plane) mirror reflection $\mathcal{B}$. For this purpose we define the normalized difference[2])

$$\Delta = \frac{W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) - W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1', \boldsymbol{k}_2')}{W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) + W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1', \boldsymbol{k}_2')}. \tag{10}$$

Since the triangle $\mathcal{B}$ can not be retrieved from $\mathcal{A}$ by any rotation operation in the $y-x$ plane, a finite Δ is not excluded by symmetry. This is in contrast to the case of a single photoelectron emission where $\Delta \equiv 0$ since in this case the triangle $\mathcal{A}$ reduces to a line and a reflection becomes equivalent to a rotation. Such a situation

[2]) If the photon is linearly polarized with a polarization vector aligned along the x or the y direction, a PDE experiment yields identical results for the electrons' configurations indicated in Fig. 1 by the triangle $\mathcal{A}$ or its mirror image $\mathcal{B}$.

is also encountered in the PDE case where the mirror reflection $\mathcal{A} \leftrightarrow \mathcal{B}$ reduces to a simple rotation if $k_1 = k_2$ or if $\hat{\boldsymbol{k}}_1$ is parallel or antiparallel to $\hat{\boldsymbol{k}}_2$. In these situations Δ vanishes.

At this stage it is decisive to note that the whole experiment as depicted in Fig.1 is invariant under spatial rotation and is parity conserving. As noted above a mirror reflection at a plane is nothing else but an inversion with respect to the origin followed by an appropriate rotation. Therefore the relation holds

$$W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1', \boldsymbol{k}_2') = W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}', \boldsymbol{k}_1, \boldsymbol{k}_2), \tag{11}$$

where $\hat{\boldsymbol{\epsilon}}'$ is the mirror image of $\hat{\boldsymbol{\epsilon}}$ with respect to the $x - z$ plane. Since we have chosen in Fig.1 $\hat{\boldsymbol{\epsilon}}$ to be $\hat{\boldsymbol{\epsilon}} = \frac{1}{\sqrt{2}}(1, +i, 0)$ we deduce that $\hat{\boldsymbol{\epsilon}}' = \frac{1}{\sqrt{2}}(1, -i, 0) = \hat{\boldsymbol{\epsilon}}^*$. Therefore we arrive at the relation

$$\Delta = \frac{W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) - W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}^*, \boldsymbol{k}_1, \boldsymbol{k}_2)}{W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) + W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}^*, \boldsymbol{k}_1, \boldsymbol{k}_2)}. \tag{12}$$

This means to measure Δ (Eq.(10)) one has to choose a configuration for $\boldsymbol{k}_1, \boldsymbol{k}_2$ such as the triangle $\mathcal{A}$ shown in Fig. 1. Then, one measures the PDE signal for left ($\hat{\boldsymbol{\epsilon}}$) and for right ($\hat{\boldsymbol{\epsilon}}^*$) hand circular polarized light and evaluates the *circular dichroism* according to Eq.(12). Such experiments has been done in Refs.[6, 7, 9, 10, 14]. Before discussing the experimental results for Δ let us point out the significance of this quantity for the symmetry of the many-body wave function.

From Eq.(9) and since $\hat{\boldsymbol{\epsilon}} = \frac{1}{\sqrt{2}}(1, +i, 0) = \frac{1}{\sqrt{2}}[(1, 0, 0) + i(0, 1, 0)] =: \frac{1}{\sqrt{2}}[\hat{\boldsymbol{\epsilon}}_x + i\hat{\boldsymbol{\epsilon}}_y]$ we can write

$$W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{C}{2}\left| t_x + it_y \right|^2 \tag{13}$$

$$W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1', \boldsymbol{k}_2') = \frac{C}{2}\left| t_x' + it_y' \right|^2 . \tag{14}$$

Here we have introduced the definitions $t_{x/y} = \hat{\boldsymbol{\epsilon}}_{x/y} \cdot \langle \Psi_{\boldsymbol{k}_1, \boldsymbol{k}_2}(\boldsymbol{r}_1, \boldsymbol{r}_2)|\boldsymbol{D}|\Phi\rangle$ and $t_{x/y}' = \hat{\boldsymbol{\epsilon}}_{x/y} \cdot \langle \Psi_{\boldsymbol{k}_1', \boldsymbol{k}_2'}(\boldsymbol{r}_1, \boldsymbol{r}_2)|\boldsymbol{D}|\Phi\rangle$. The matrix elements t_x and t_y can be interpreted as the PDE dipole transition amplitudes for the electrons' configuration as shown in Fig.1 with the photon being linear polarized and its electric field is oscillating along the x or the y direction, respectively. It is readily clear from symmetry arguments that $|t_x'| = |t_x|$ and $|t_y'| = |t_y|$. However the *phases* of $t_{x/y}'$ are generally *different* from those for $t_{x/y}$ (cf. also [26]). Therefore, it follows from Eqs.(10,13,14) that Δ has to be related to some phase relations that need to be uncovered. To do that we write $t_{x/y} = |t_{x/y}|e^{i\varphi_{x/y}}$ and $t_{x/y}' = |t_{x/y}'|e^{i\varphi_{x/y}'}$ and deduce from simple algebraic manipulations that

$$W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) - W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1', \boldsymbol{k}_2') = C|t_x|\,|t_y|\,(\sin\phi' - \sin\phi)\,. \tag{15}$$

Here we introduced the phase differences $\phi' = \varphi_y' - \varphi_x'$, $\phi = \varphi_y - \varphi_x$. Equivalently one can show that [29]

$$W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) + W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}^*, \boldsymbol{k}_1, \boldsymbol{k}_2) = C|t_x|^2 + |t_y|^2 \tag{16}$$

$$W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}, \boldsymbol{k}_1, \boldsymbol{k}_2) - W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}^*, \boldsymbol{k}_1, \boldsymbol{k}_2) = -2C|t_x|\,|t_y|\sin\phi \tag{17}$$

Recalling Eq.(11) we deduce from Eqs.(15,17) the relation

$$\phi' = -\phi + 2\pi n, \tag{18}$$

where n is an integer number. The question is now whether one can measure ϕ (and hence ϕ'). The answer is provided by Eqs.(12, 16,17) which imply that

$$\sin\phi = -\Delta \frac{W(\hat{\boldsymbol{\epsilon}}_x, \boldsymbol{k}_1, \boldsymbol{k}_2) + W(\hat{\boldsymbol{\epsilon}}_y, \boldsymbol{k}_1, \boldsymbol{k}_2)}{\sqrt{W(\hat{\boldsymbol{\epsilon}}_x, \boldsymbol{k}_1, \boldsymbol{k}_2)\, W(\hat{\boldsymbol{\epsilon}}_y, \boldsymbol{k}_1, \boldsymbol{k}_2)}}, \tag{19}$$

where $W(\hat{\boldsymbol{\epsilon}}_x, \boldsymbol{k}_1, \boldsymbol{k}_2) = C|t_x|^2$ and $W(\hat{\boldsymbol{\epsilon}}_y, \boldsymbol{k}_1, \boldsymbol{k}_2) = C|t_y|^2$ are the PDE cross sections that are measurable by utilizing linear polarized light (note that $W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}_{x/y}, \boldsymbol{k}_1, \boldsymbol{k}_2) \equiv W_{\mathcal{B}}(\hat{\boldsymbol{\epsilon}}_{x/y}, \boldsymbol{k}'_1, \boldsymbol{k}'_2)$). In addition, it is readily deduced that $W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}_y, \boldsymbol{k}_1, \boldsymbol{k}_2) = W_{\mathcal{A}}(\hat{\boldsymbol{\epsilon}}_x, \tilde{\boldsymbol{k}}_1, \tilde{\boldsymbol{k}}_2)$ where $\tilde{\boldsymbol{k}}_1$ and $\tilde{\boldsymbol{k}}_2$ are obtained from $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ via a $\pi/2$ rotation with respect to the z axis (i.e. instead of a $\pi/2$ rotation of the polarization vector from $\hat{\boldsymbol{\epsilon}}_x$ to $\hat{\boldsymbol{\epsilon}}_y$ one can leave $\hat{\boldsymbol{\epsilon}}_x$ fixed and perform a $\pi/2$ rotation of the momenta $\boldsymbol{k}_1,\ \boldsymbol{k}_2$).

3.1 Numerical results and comparison with experiments

To sense by means of PDE the difference between the two-electron states described by the triangles $\mathcal{A}$ and $\mathcal{B}$ (as depicted Fig.1) one needs to perform PDE measurements with circular polarized light, as evident from Eqs.(10,12). For a measurement of the phase differences ϕ, ϕ' one needs in addition a PDE experiment with linear polarized light, as required by Eq.(19). These kind of experimental studies have been conducted using a ground-state He atom. Fig. 2 shows a typical result of these experiments for the configuration of Fig.1. As obvious from Fig.2(b), the difference between the states quantified by $\mathcal{A}$ and $\mathcal{B}$ shows up in a finite Δ in Fig.2(b) with remarkable angular variations. Δ vanishes at $\theta_2 = 0°, 180°, 360°$ as in this case $\mathcal{A}$ and $\mathcal{B}$ reduce to a line and become equivalent (θ_2 is the angular position of $\boldsymbol{k}_2$ with respect to the x axis). The actual magnitude and the shape of Δ depend on the details of the many-body wave function and can not be explained by geometrical arguments. For example, if one employs plane waves for the two escaping electrons Δ vanishes identically, as shown below.

As explained in the previous section, Δ can also be employed to deduce the phase difference ϕ according to Eq.(19). This is done in Fig.2(c). We notice from this figure a significant angular variation of the phase difference ϕ. The sign of ϕ is solely determined by Δ (cf. Eq.(19), i.e. it is determined by the influence of the mirror reflection (cf. Eqs.(10,12)). The value of $\phi(\theta_2)$ is small when $\Delta(\theta_2)$ is small (cf. Eq.(19)) and hence the resemblance between the minima and the maxima in the angular distributions of $\phi(\theta_2)$ and $\Delta(\theta_2)$, as observed in Figs.2(c,b).

To uncover the significance of the phase difference ϕ for the wave function Ψ we recall that ϕ has been introduced as a difference of the phases $\varphi_{x/y}$of the transition amplitudes $t_{x/y}$. The relation to the phase of the wave function Ψ becomes clear

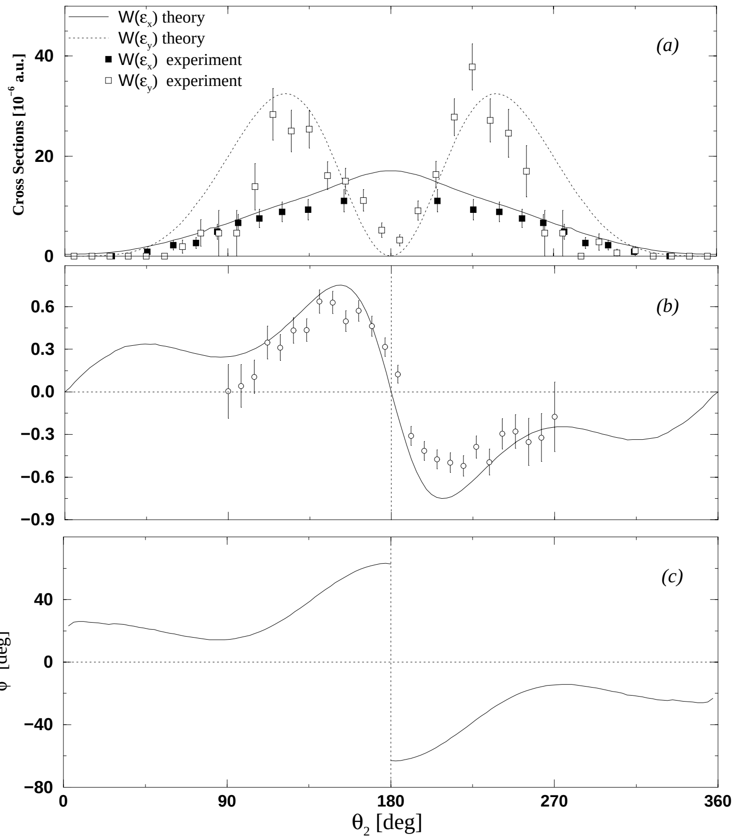

Fig. 2. **(a)**: The cross sections for double ionization of He($^1S^e$) with a linear polarized photon in the geometry shown in Fig. 1. Two experiments are shown: In the first one (solid line, labeled $W(\boldsymbol{\epsilon}_x)$) the photon's polarization vector $\hat{\boldsymbol{\epsilon}}$ is fixed along the x direction whereas in the second case (dotted line, labeled $W(\boldsymbol{\epsilon}_x)$) the polarization vector $\hat{\boldsymbol{\epsilon}}$ is along the y direction. The excess energy is 20 eV. As shown in Fig.1, both electrons are detected in the $x - y$ plane. One fast electron (electron 1 with 17.5 eV) is detected along the x-direction whereas the angular distribution of the slower one (electron 2) is scanned as function of θ_2 where θ_2 determines the angular position of $\hat{\boldsymbol{k}}_2$ with respect to x axis (cf. Fig. 1). Experimental data are due to Ref.[27]. The initial state has been modeled by a three-parameter Hylleraas wave function [8, 28] whereas the same correlated final state has been employed as in Ref.[7]. The velocity form has been employed. **(b)**: The same experimental arrangement of the electrons' as in Fig.1 (and in (**a**)), however, the experiment is performed with circularly polarized photons with the photon wave vector $\boldsymbol{k}$ pointing along the z direction (cf. Fig.1). From the measurements with left and right circular polarized photons one determines the circular dichroism Δ according to Eq.(12). The solid curve is the theoretical prediction using the same model as in (**a**). The absolute experimental data are due to Refs.[7, 14]. **(c)**: The difference $\phi' = -\phi$ as derived according to Eq.(19) and using the calculations shown in (**a**) and (**b**).

when we write $t_{x/y}$ in a momentum space representation and in the velocity form:

$$t_{x/y} \propto \langle \Psi_{\boldsymbol{k}_1,\boldsymbol{k}_2} | \hat{\boldsymbol{\epsilon}}_{x/y} \cdot (\boldsymbol{\nabla}_1 + \boldsymbol{\nabla}_2) | \Phi \rangle = i\hat{\boldsymbol{\epsilon}}_{x/y} \cdot \int (\boldsymbol{p} + \boldsymbol{q}) \; \tilde{\Psi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{p},\boldsymbol{q}) \tilde{\Phi}(\boldsymbol{p},\boldsymbol{q}) d^3\boldsymbol{p}\, d^3\boldsymbol{q}, \tag{20}$$

where $\tilde{\Phi}(\boldsymbol{p},\boldsymbol{q})$ is the (six-dimensional) Fourier transform of the initial state. Since we assumed the initial state to be randomly oriented the Fourier transform is real. Therefore, the only phase contribution to the integral (20) stems from the phase $\phi_\Psi = \phi_\Psi(\boldsymbol{k}_1, \boldsymbol{k}_2; \boldsymbol{p}, \boldsymbol{q})$ of the momentum-space wave function $\tilde{\Psi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{p},\boldsymbol{q}) = |\tilde{\Psi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{p},\boldsymbol{q})| \; e^{i\phi_\Psi}$. The direct link between the transition-amplitude phases $\phi_{x/y}$ and wave-function phase ϕ_Ψ becomes transparent if we employ the peaking approximation $\tilde{\Psi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{p},\boldsymbol{q}) \approx \tilde{\xi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}(\boldsymbol{p},\boldsymbol{q}) \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1) \delta^{(3)}(\boldsymbol{q} - \boldsymbol{k}_2)$. In this case we obtain for the experimental arrangement of Fig.1 $\phi = \phi_\xi(\theta_1 - \pi/2, \theta_2 - \pi/2) - \phi_\xi(\theta_1, \theta_2)$ and $\phi' = -\phi$. Here ϕ_ξ is the phase of the function $\tilde{\xi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}$. If the continuum states of the two electrons are modeled by plane waves the peaking approximation becomes exact and $\tilde{\xi}^*_{\boldsymbol{k}_1,\boldsymbol{k}_2}$ is a mere constant. This leads to $\phi_\xi \equiv 0$, which in turn means $\phi \equiv 0$.

4 Conclusions

In this paper we gave a brief account on how symmetry properties and internal phase relations of an interacting three-body wave function can be investigated by means of photons. In particular we pointed out that mirror-reflection symmetries can be tested by utilizing circular photons. In this case the helicity of the photon is transferred to the few-body system. The physical properties of this excited helical system is strongly dependent on whether the helicity is positive of negative. It has been shown that this statement can be tested experimentally by determining the continuum spectra of an interacting electron pair with a positive (or a negative) chirality created upon the double excitation of a random atom by a circular photon with the appropriate helicity. The dependence of these spectra on the helicity of the exciting photon provides an experimental tool to investigate the internal phase relations within the three-body wave function.

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