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# Localization effects in magnetic structures with spin–orbit interaction

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**Abstract**

Localization corrections to electrical and spin conductivities of bulk and thin-film ferromagnetic systems are analyzed theoretically in the case of electron spin–orbit scattering from impurities. The effect of internal magnetic field on the corrections is studied in detail. It is shown, that the magnetoresistance of ferromagnetic systems, related to the localization corrections, is always negative. 

*Keywords:* Magnetoresistance; Conductivity; Disordered systems; Spin–orbit interaction

Recent interest in spin polarized transport in magnetic metallic and semiconductor structures follows from expected applications in magnetoelectronics [1–3]. Although the room temperature regime is more attractive for applications, the low-temperature regime seems to be more interesting from the basic research point of view. This is due to various quantum phenomena which lead to pronounced and measurable effects in transport characteristics. One of such phenomena is the weak localization (or quantum interference of electron waves scattered by defects), which gives rise to anomalous temperature and magnetic field dependences of electrical conductivity at sufficiently low temperatures.

The quantum corrections to conductivity in non-magnetic metals and semiconductors were extensively studied in the past two decades [4–6]. As we show in this paper, these effects can be observed in ferromagnets too, but in this case they have some peculiarities following from strong internal exchange field. Some of the peculiarities have been discussed in Ref. [7], particularly those due to electron–electron interaction. The issue of weak localization in ferromagnets is a subtle problem, because it is known that the localization corrections in non-magnetic systems are suppressed by a sufficiently large magnetic induction $B$. One might then expect a similar suppression of weak localization by an internal magnetic induction $B_{\rm int}$ in ferromagnets. However, the localization corrections have been observed in Ni [8]. It is then reasonable to assume that the internal magnetic induction existing inside the ferromagnets can reduce the localization corrections instead of destroying them totally. Very likely, one can expect only a weak effect of $B_{\rm int}$ in the case of novel magnetic semiconductors like GaMnAs alloys [3].

Spin–orbit (SO) scattering in non-magnetic metals is known to have a strong effect on the quantum corrections. It can even reverse the sign of the localization correction (antilocalization), which results in a positive magnetoresistance at weak magnetic fields [9,10]. However, the situation in ferromagnetic systems is quite different. The processes leading to antilocalization in non-magnetic systems are totally suppressed in ferromagnets, which results in a negative magnetoresistance. On the other hand, the SO interaction controls the magnitude of the localization correction, and the interplay between an effective SO relaxation time, phase relaxation time, and a characteristic time related to

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the magnetic induction, determines the dependence of electrical conductivity on the external magnetic field and also the dependence on temperature.

We consider the following Hamiltonian of a ferromagnet with SO scattering:

$$H = \int \mathrm{d}^3\mathbf{r}\,\psi^{\dagger}(\mathbf{r})\left[-\frac{\nabla^2}{2m} - M\sigma_z + V(\mathbf{r})\right]\psi(\boldsymbol{r}), \tag{1}$$

where the axis $z$ is assumed to be along the magnetization $\mathbf{M}$, $\psi$ is a spinor field, and we put $\hbar = 1$. In the presence of a magnetic induction $\mathbf{B} = \mathrm{rot}\,\mathbf{A}$, the gradient operator $\nabla$ is replaced by $\nabla - \mathrm{i}\,e\mathbf{A}/c$.

The random potential $V(\mathbf{r})$ of impurities consists of two components: a spin-independent potential $V_0(\mathbf{r})$, and the SO component $V_{\mathrm{so}}(\mathbf{r})$. Matrix elements of the latter component have the form $(V_{\mathrm{so}})_{\mathbf{k}\alpha,\mathbf{k}'\beta} = \mathrm{i}\,V_1(\mathbf{k}\times\mathbf{k}')\cdot\sigma_{\alpha\beta}$ for the transitions $(\mathbf{k},\alpha)\to(\mathbf{k}',\beta)$, where $V_1$ is a constant, $\mathbf{k}$ and $\mathbf{k}'$ are the initial and final electron wavevectors, $\alpha$ and $\beta$ describe the corresponding spin states, and $\sigma = (\sigma_x,\sigma_y,\sigma_z)$ are the Pauli matrices.

The key element of the weak localization theory is the Cooperon [4–6] which can be presented by a ladder in the particle–particle channel with two propagators describing electrons with small total momentum and close energy parameters. In the case of ferromagnets and as long as $M\tau_\sigma \gg 1$, where $\tau_\sigma$ is the momentum relaxation time of electrons with spin $\sigma$ ($\sigma = \uparrow,\downarrow$), this channel does not include ladder elements with the Green functions corresponding to the opposite spin orientations. Using the standard method of calculating the Cooperon [4,5], we solved the ladder equations for spin-up and spin-down channels, taking into account spin–flip processes due to SO interaction. We also calculated the self-energy for spin-up and spin-down electrons, taking into account both potential and SO scattering terms. The results of these calculations can be summarized as follows.

In the 3D case (bulk ferromagnet), the total electron relaxation time $\tau_\uparrow$ is determined by

$$\frac{1}{\tau_\uparrow} = \frac{1}{\tau_{0\uparrow}} + \frac{1}{\tau^z_{\mathrm{so}\uparrow}} + \frac{2\nu_\downarrow}{\nu_\uparrow}\frac{1}{\tau^x_{\mathrm{so}\uparrow}}, \tag{2}$$

and a similar formula also holds for $\tau_\downarrow$. Here, $\tau_{0\sigma}$, $\tau^z_{\mathrm{so},\sigma}$, and $\tau^x_{\mathrm{so},\sigma}$ are the relaxation times related to the potential scattering, non-spin–flip SO scattering, and spin–flip SO scattering, respectively, whereas $\nu_\uparrow$ and $\nu_\downarrow$ are the densities of states for spin-up and spin-down electrons.

The effective relaxation time of the Cooperon, which determines the localization correction to conductivity, is given by

$$\frac{1}{\tilde{\tau}_{\mathrm{so}\uparrow}} = 2\left(\frac{1}{\tau^z_{\mathrm{so}\uparrow}} + \frac{\nu_\downarrow}{\nu_\uparrow}\frac{1}{\tau^x_{\mathrm{so}\uparrow}}\right). \tag{3}$$

On the other hand, the localization correction $\Delta\sigma$ to the static conductivity is determined by the loop diagrams including the Cooperon [5,4]. In a 3D case we obtain

$$\Delta\sigma^{3D} = C + \frac{e^2}{4\pi^2}\sum_{\sigma=\uparrow,\downarrow}\frac{1}{D_\sigma^{1/2}}\left(\frac{1}{\tilde{\tau}_{\mathrm{so},\sigma}} + \frac{1}{\tau_{\varphi,\sigma}}\right)^{1/2}, \tag{4}$$

which is a simple generalization of the known formula for the quantum correction in non-magnetic systems. In the above formula $\tau_{\varphi,\sigma}$ is the phase relaxation time related to inelastic scattering processes [5,4], $D_\sigma$ is the spin dependent diffusion constant, and the constant $C$ can be roughly estimated as

$$C \simeq -\frac{e^2}{4\pi^2}\left[(D_\uparrow\tau_\uparrow)^{-1/2} + (D_\downarrow\tau_\downarrow)^{-1/2}\right]. \tag{5}$$

According to Eqs. (4) and (5), the localization correction is negative. By decreasing $\tilde{\tau}_{\mathrm{so}}$ and/or $\tau_\varphi$, we suppress the correction, increasing this way the conductivity.

As is known, the magnetic induction suppresses the localization correction to conductivity. If the total magnetic induction is $B$, then for $\delta\sigma(B) \equiv \Delta\sigma(B) - \Delta\sigma(0)$ we find in a 3D case the following formula [11]:

$$\delta\sigma^{3D}(B) = -\frac{e^2}{16\pi^2 l_B}\sum_{n=0}^{\infty}\sum_{\sigma=\uparrow,\downarrow}\left[\frac{1}{\sqrt{n+\frac{1}{2}+\delta_\sigma}} - 2\sqrt{n+1+\delta_\sigma} + 2\sqrt{n+\delta_\sigma}\right], \tag{6}$$

where $l_B = (c/eB)^{1/2}$ is the magnetic length, $c$ is the light velocity, and $\delta_\sigma = \frac{1}{4}(l_B^2/D_\sigma)\,(1/\tilde{\tau}_{\mathrm{so},\sigma} + 1/\tau_{\varphi,\sigma})$. As follows from Eq. (6), $\delta\sigma^{3D}(B)$ is positive and its magnitude increases with increasing magnetic field. This means that one finds a negative magnetoresistance, in spite of the presence of SO interaction.

In the case of magnetic films, the weak localization corrections are effectively 2D when the film thickness $d$ is smaller than the characteristic length $L_0 = [(D\tilde{\tau}_{\mathrm{so}})^{-1} + (D\tau_\varphi)^{-1}]^{-1/2}$, and the effect of perpendicular magnetic induction has the following form [4,5]

$$\Delta\sigma^{2D}(B) = -\frac{e^2}{4\pi^2}\sum_{\sigma=\uparrow,\downarrow}\left[\psi\left(\frac{1}{2}+\frac{\tau_{B\sigma}}{\tau_\sigma}\right) - \psi\left(\frac{1}{2}+\frac{\tau_{B\sigma}}{\tilde{\tau}_{\mathrm{so},\sigma}}+\frac{\tau_{B\sigma}}{\tau_{\varphi,\sigma}}\right)\right], \tag{7}$$

where $1/\tau_{B\sigma} = 4eBD_\sigma/c$ and $\psi(x)$ is the digamma function.

One can also find the localization corrections to the spin conductivity, defined as a spin current in response to an electric field. In this case, the spin current is the difference of currents in the spin-up and spin-down channels, and for the quantum correction we obtain

$$\Delta\sigma^{3D}_{\mathrm{spin}} = \frac{e}{4\pi^2}\left[\frac{1}{D_\uparrow^{1/2}}\sqrt{\frac{1}{\tilde{\tau}_{\mathrm{so}\uparrow}}+\frac{1}{\tau_{\varphi\uparrow}}} - \frac{1}{D_\downarrow^{1/2}}\sqrt{\frac{1}{\tilde{\tau}_{\mathrm{so}\downarrow}}+\frac{1}{\tau_{\varphi\downarrow}}}\right] \tag{8}$$

in a 3D case, and

$$\Delta\sigma_{\rm spin}^{2D} = -\frac{e}{4\pi^2}\ln\left[\frac{D_\uparrow\left(\tilde{\tau}_{{\rm so}\downarrow}^{-1}+\tau_{\varphi\downarrow}^{-1}\right)}{D_\downarrow\left(\tilde{\tau}_{{\rm so}\uparrow}^{-1}+\tau_{\varphi\uparrow}^{-1}\right)}\right] \tag{9}$$

in an effectively 2D case. The main contributions to the correction, originated from the shortest time $\tau_\sigma$, are exactly canceled. Thus, the correction to the spin conductivity is determined by the SO scattering and the phase relaxation.

In our description, we have taken into account the effect of magnetic induction **B**, which enters the kinetic energy of electrons through the vector potential **A** and leads to the suppression of localization corrections, Eqs. (6) and (7). The total magnetic induction **B** inside a ferromagnet includes the external part $\mathbf{B}_{\rm ext}$ and the internal magnetic induction $\mathbf{B}_{\rm int}$. In strong ferromagnets the magnitude of $B_{\rm int}$ can be rather high. For example, in bulk Fe one finds $B_{\rm int}\simeq 2\,{\rm T}$. Our estimations show that for not very pure Fe with electron mean free path $l\simeq 10^{-6}\,{\rm cm}$, the critical induction, which totally suppresses the localization corrections, is $B_{\rm crit}\simeq 7\,{\rm T}$. Therefore, the localization corrections can be observed. In the case of thin magnetic films, the demagnetizing factor is of crucial importance. For example, when the magnetization $\mathbf{M}_0$ is perpendicular to the film, the demagnetizing factor is equal to 1, and we have $\mathbf{B}_{\rm int}=0$. In this case, our results can be applied with the magnetic induction $B$ equal to the external magnetic field $B_{\rm ext}$.

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