Micromagnetic localization

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The localization of nucleation modes in inhomogeneous ferromagnets and its influence on the coercivity are investigated. From the formal analogy between quantum mechanics and micromagnetics follows that anisotropy inhomogeneities may cause localization. The nucleation modes of one-dimensional arrays, such as multilayers composed of hard and soft magnetic materials, are localized even if the superlattice exhibits a nearly ideal periodicity. Gaussian distributions of the layer thicknesses lead to Urbach tails and very low coercivities, but a maximum thickness l_m of the soft layers suppresses the Urbach tails. The related problem of magnetic viscosity leads to a supersymmetric Fokker–Planck description where the time dependence of the magnetization is given by the ground-state mode of a fermionic potential. © *1998 American Institute of Physics.* [S0021-8979(98)50911-5]

I. INTRODUCTION

Localization means that the eigenfunctions of a partial differential equation are concentrated in a small volume. A well-known problem is electron localization: metallic wave functions, such as free-electron plane waves, are delocalized, whereas electrostatic correlations and disorder may give rise to Mott and Anderson localization, respectively.[1–3] Mott localization is a many-body effect and occurs, for example, if the interatomic distance of a metal exceeds a threshold above which metallic conductivity vanishes. Here we are concerned with the Anderson localization in a random potential.

As discussed for example in Ref. 4, there is a formal analogy between micromagnetics and quantum mechanics. Nuclei in homogeneous ellipsoids of revolution are delocalized, but localization may be caused by magnetic inhomogeneities. This micromagnetic localization is of practical importance because it determines the nucleation of reverse domains and therefore affects the coercivity. An example is oriented nanostructured two-phase permanent magnets such as $Nd_2Fe_{14}B/Fe$, where very high energy products are expected.[4–7] In these structures, the rare-earth-containing hard regions act as a skeleton which stabilize the high magnetization of the soft phase, but nucleation modes localized in extended soft regions tend to destroy coercivity.

A related problem is the time dependence of quantities such as the remanent magnetization (magnetic viscosity). On an atomic level, magnetic viscosity arises from the interaction of the magnetic moments with other degrees of freedom such as lattice vibrations. As emphasized in Ref. 8, the heat bath associated with the nonmagnetic degrees of freedom leads to a Fokker–Planck diffusion of the magnetic moments in the zero-temperature potential E_m. However, even for one-dimensional problems such as the motion of a domain wall in a disordered potential there exists no exact solution.

Here we present an interpretation of micromagnetics in terms of the localization problem. Particular emphasis is put on nucleation modes and long-time magnetic relaxations.

II. LOCALIZATION OF NUCLEATION MODES

For simplicity, we will restrict ourselves to the energy functional

$$E_m = \int \left(A \frac{\nabla \mathbf{M}^2}{M_s^2} + K_1(\mathbf{r}) \frac{M_z^2}{M_s^2} - \mu_0 M_z H \right) d\mathbf{r}, \tag{1}$$

where A is the exchange stiffness and $K_1(\mathbf{r})$ denotes the lowest-order uniaxial anisotropy constant. The magnetostatic self-interaction is approximated by a demagnetizing field, because anisotropy fields $2K_1/\mu_0 M_s$ tend to be much larger than stray fields in hard magnets such as ultrathin films and rare-earth permanent magnets.[4,9,10] Typical microstructures of interest are shown in Fig. 1.

To obtain nucleation modes we rewrite $\mathbf{M}$ as

$$\mathbf{M}(\mathbf{r}) = M_s \sqrt{1 - m(\mathbf{r})^2}\, \mathbf{e}_z + M_s \mathbf{m}(\mathbf{r}) \tag{2}$$

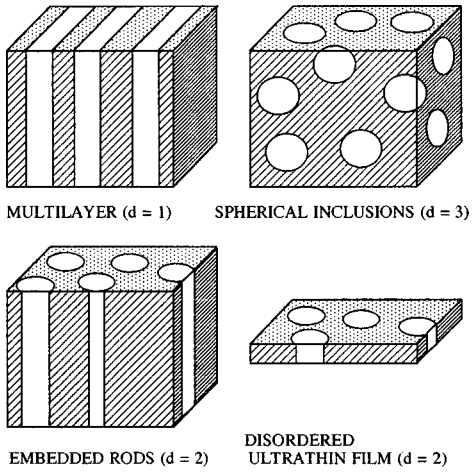

FIG. 1. Inhomogeneous structures consisting of magnetically hard (dark) and soft (white) regions. The orientation of the common easy axis is irrelevant as long as it is parallel to the applied magnetic field.

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and expand E_m into powers of the small transverse magnetization component $\mathbf{m}=m_x\mathbf{e}_x+m_y\mathbf{e}_y$. Minimizing E_m then yields

$$-A\nabla^2\mathbf{m}+2K_1(\mathbf{r})\mathbf{m}=-\mu_0 M_s H\mathbf{m}. \tag{3}$$

This equation is degenerate with respect to m_x and m_y, so that we restrict ourselves to any direction in the xy plane. In practice, small deviations from the common c-axis anisotropy (grain misalignment) and magnetostatic interactions break the symmetry and fix the direction of $\mathbf{m}$.

Equation (3) is reminiscent of Schrödinger's equation for an electron in an electrostatic potential V. In *this quantum-mechanical analogy*, A, K_1, and $-\mu_0 M_s H/2$ are analogous to $\hbar^2/m_e$, V, and E, respectively. The ground-state energy E_0 corresponds to the *nucleation field* $H=-H_N$, which determines the coercivity of nucleation–controlled magnets.[4] In the ordered limit, Eq. (3) has been solved for a number of cases.[4,11–13]

Lowest-order perturbation theory yields[4,5,9]

$$H_N=\frac{2\langle K_1(\mathbf{r})\rangle_v}{\mu_0 M_s} \tag{4}$$

so that the nucleation field is given by the volume-averaged anisotropy constant $\langle K_1(\mathbf{r})\rangle_v=K$. In the quantum-mechanical analogy, this approach is known as the *virtual crystal* approximation.[14]

There are various methods to solve the random-potential band structure problem.[3,14] Here we restrict ourselves to *second-order perturbation theory*. Applying the quantum-mechanical expression

$$E=E_0+\langle\psi_0|V|\psi_0\rangle-\sum_k \frac{|\langle\psi_k|V|\psi_0\rangle|^2}{E_k-E_0} \tag{5}$$

to Eq. (3) yields

$$\mu_0 H_N=\frac{2K}{M_s}-\frac{4}{(2\pi)^d A M_s}\int \frac{1}{\mathbf{k}^2}\,G(\mathbf{k})\,d^d\mathbf{k}. \tag{6}$$

Here $G(\mathbf{k})=\int \exp(i\mathbf{k}.\mathbf{r})\cdot\langle[K_1(\mathbf{r})-K][K_1(0)-K]\rangle_v d\mathbf{r}$ is the Fourier-transformed autocorrelation function of the disorder. For example, the isotropic distribution $\langle[K_1(\mathbf{r})-K]\times[K_1(0)-K]\rangle_v=K_0^2\exp(-r^2/2R^2)$ yields $G(\mathbf{k})=(2\pi R^2)^{d/2}K_0^2\exp(-k^2R^2/2)$. In these equations, R is the average radius of the hard and soft regions and $K_0=K_h\sqrt{f_s(1-f_s)}$, where K_h is the anisotropy constant of the hard phase and f_s is the volume fraction of the soft phase.

Localization depends on the *dimensionality* of the problem and is most pronounced in one and two dimensions.[1,14] Figure 1 shows some one-, two-, and three-dimensional structures of interest. In Eq. (6), the $1/\mathbf{k}^2$ term causes the corrections to diverge in less than two dimensions. This result is related to the absence of metallic conduction in less than two dimensions.[3,14] For $d>2$, Eq. (6) yields

$$\mu_0 H_N=\frac{2K}{M_s}-\frac{4R^2}{(d-2)AM_s}K_0^2. \tag{7}$$

The $1/(d-2)$ dependence in this equation shows that three-dimensional configurations of soft and hard regions are not very much affected by minor inhomogeneities. As a rule, coercivity breaks down if the size of the soft regions is larger than the domain-wall width $\pi\sqrt{A/K_h}\approx 4$ nm of the hard phase (compare also Refs. 4 and 9).

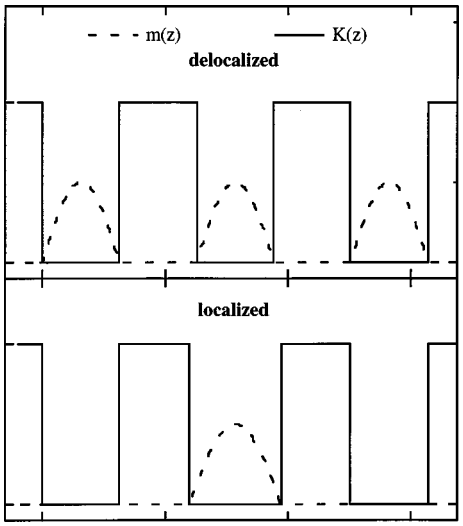

FIG. 2. Nucleation modes in (a) ideally periodic and (b) nearly periodic multilayers. The solid lines show the K_1 profiles in the z direction, while the nucleation modes $|\mathbf{m}(z)|$ are given by the dashed lines.

In one dimension, for example in multilayers, arbitrary small disorder leads to localization. Figure 2 compares delocalized and localized nucleation modes $\mathbf{m}(z)$ in (a) periodic and (b) nearly periodic multilayers. In Fig. 2(b), one soft layer is thicker by about 15% than the others, and the nucleation mode is localized. As in quantum mechanics, there is a small resonance interaction (tunneling) between the potential minima,[4] but in fair approximation this contribution can be neglected here and following Ref. 4 we estimate that the nucleation field of Fig. 2(b) is smaller by about 30% than that of the periodic lattice Fig. 2(a).

In most cases, disorder leads to extended soft regions which destroy coercivity. In the context of electron localization, the low-lying states responsible for this behavior are known as *Urbach tails*, and asymptotically the density of states of the Urbach tail is given by the probability distribution of the structural disorder.[3] In multilayers, a Gaussian distribution of thicknesses l_s of the soft layers yields a logarithmic dependence of the nucleation field on the total film thickness t and yields $H_N=0$ for $t\to\infty$.[18] However, if the thicknesses obey $l_s\leqslant l_m$ then the Urbach tails are cut off and the nucleation field scales as $1/l_m^2$, as sketched for example in Refs. 4 and 13.

III. SUPERSYMMETRY

An atomic approach towards magnetic viscosity is to consider random thermal forces $\xi(t)$ acting on the magnetization vector. For simplicity, we will restrict ourselves to a single magnetization degree of freedom s. Examples are $s=\sin\theta$ and $s=x$ in fine-particle and pinning-type magnets, respectively. This leads to the *magnetic Langevin equation*

$$\frac{\partial s}{\partial t}=-\frac{\Gamma_0}{k_B T}\frac{\partial E_m}{\partial s}+\sqrt{2\Gamma_0}\,\xi(t), \tag{8}$$

where $\Gamma_0 = 1/\tau_0$ is an atomic attempt frequency.[8,15,16] The random forces obey $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$, where $\delta(x)$ is the delta or ''needle'' function defined by $\delta(x) = 0$ for $x \neq 0$ and $\int \delta(x)dx = 1$. At low temperatures, the $\partial s/\partial t$ and ξ terms are negligible and Eq. (8) reduces to the trivial minimization problem $\partial E_m/\partial s = 0$.

The probability distribution $P(s,t)$ obeys the magnetic *Fokker–Planck equation*[8,15]

$$\Gamma_0^{-1} \partial P/\partial t = (k_B T)^{-1} \partial (P \partial E_m/\partial s)/\partial s + \partial P^2/ds^2. \qquad (9)$$

As Eq. (1), the Fokker–Planck equation implies that macroscopic magnetization jumps consist of a chain of microscopic events. A simple one-dimensional example are small patches of (111) transition-metal films with easy-plane anisotropy but without in-plane anisotropy, that is $E_m(\phi) = \text{const}$. For the initial condition $\mathbf{M} = \mathbf{M}_s \mathbf{e}_x$ we obtain $P(\phi,t) = (4\pi\Gamma_0 t)^{-1/2} \exp(-\phi^2/4\Gamma_0 t)$ and $\langle \cos \phi \rangle = \exp(-\Gamma_0 t)$. There is, however, no general solution of the one-dimensional Fokker–Planck equation.[8,15]

In equilibrium, where $\partial P/\partial t = 0$, Eq. (9) yields the relaxation rate $\Gamma = 0$ and $P(s) = Z^{-1} \exp(-E_m/k_B T)$. However, to understand the long-time magnetic-viscosity limit we have to consider the smallest nonzero relaxation rate $\Gamma_1 = 1/\tau_1$. A conceptionally very simple solution of this problem is provided in terms of supersymmetric quantum mechanics, which unifies bosonic and fermionic properties of matter. The observed particle masses indicate a strong breaking of the supersymmetry in elementary particle physics, but the concept is a useful idea not only in elementary particle physics but also in solid-state physics.[15,17] The formal ansatz $P(s,t) = \exp(-\Gamma t)\exp(-E_m/2k_B T)\Psi(s)$ transforms Eq. (9) into

$$\frac{\Gamma}{\Gamma_0} \Psi = -\frac{\partial^2 \Psi}{\partial s^2} + V_+ \Psi, \qquad (10)$$

where the so-called bosonic potential V_+ and its fermionic counterpart V_- are given by

$$V_\pm = (\partial E_m/\partial s)^2/4k_B^2 T^2 \mp (\partial E_m^2/\partial s^2)/2k_B T. \qquad (11)$$

In supersymmetric quantum mechanics, replacing V_+ by V_- transforms the ''bosonic'' differential Eq. (10) into a fermionic equation. Since the first excited eigenvalue of the bosonic problem is equal to the lowest eigenvalue for the fermionic potential,[17] the long-time limit of magnetic viscosity is a ground-state property of the fermionic problem. However, the localization behavior of the fermionic ground-state mode is more complicated than that shown in Fig. 2 and requires further analysis.

IV. DISCUSSION AND CONCLUSIONS

In conclusion, we have analyzed the localization behavior of nucleation and magnetic-viscosity modes in terms of the new concepts of micromagnetic Urbach tails and supersymmetric magnetic viscosity. Nucleation modes in *one*-dimensional structures, such as multilayers, are localized, even if the structure is nearly periodic. Gaussian disorder destroys coercivity, but a maximum thickness $l_{\max}$ of the soft-magnetic layers achieved by careful processing assures a finite nucleation field. On the other hand, we have shown that the long-time limit of magnetic viscosity is equivalent to the ground-state localization in a fermionic supersymmetric potential.

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