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Weak localization corrections to the anomalous Hall effect

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Abstract

We calculate localization corrections to the anomalous Hall conductivity in the framework of side-jump and skew scattering mechanisms. In contrast to the ordinary Hall effect, there exists a nonvanishing localization correction to the anomalous Hall resistivity. The correction to the anomalous Hall conductivity vanishes in case of side-jump mechanism, but is nonzero for the skew scattering.

Keywords: Hall effect; Conductivity tensor; Disordered systems; Spin-orbit interaction

Recently, there has been a growing interest in the anomalous Hall effect (AHE) due to the importance of the spin polarization and spin–orbit (SO) interaction for transport properties of materials and structures of spin electronics [1,2]. Besides, the measurement of AHE is proved to be a useful tool to investigate the magnetism of layered structures [2]. Usually, two relevant mechanisms of AHE are distinguished—a skew scattering [3] and a side-jump effect [4,5]. It was shown [6] that for the skew scattering, there are non-vanishing localization corrections and vanishing interaction corrections to the off-diagonal AHE conductivity σ_{xy}. The experiments on amorphous Fe films [7] confirmed the absence of quantum corrections to σ_{xy}, which has been interpreted as a dominate role of interaction effects. Here we present the calculation of localization corrections in the case of the side-jump mechanism and analyse different approaches to the problem.

We consider the Hamiltonian of a ferromagnet with a strong exchange field **M** oriented along the axis z, and SO relativistic term (we put $\hbar = 1$)

$$H = -\frac{\nabla^2}{2m^*} - M\sigma_z - \frac{\mathrm{i}\,\lambda_0^2}{4}(\sigma \times \nabla V)\cdot\nabla + V, \qquad (1)$$

where m^* is the electron effective mass, λ_0 is a constant, which measures the strength of the SO interaction, $V(\mathbf{r})$ is a random potential created by impurities or defects, and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. The constant λ_0 has the dimensionality of length and for non-relativistic electrons in vacuum $\lambda_0 = \lambda_c/2\pi$, where $\lambda_c = 2\pi/m_0 c$ is the Compton wavelength of electron and m_0 is the free electron mass.

We assume that the potential $V(\mathbf{r})$ is short-ranged, with zero mean value, $\langle V(\mathbf{r})\rangle = 0$, where the angle brackets mean the configurational averaging over all realizations of $V(\mathbf{r})$. We shall characterize this potential by its second, γ_2, and third, γ_3, momenta, denoting $\langle V(\mathbf{r}_1)\,V(\mathbf{r}_2)\rangle = \gamma_2\,\delta(\mathbf{r}_1 - \mathbf{r}_2)$ and $\langle V(\mathbf{r}_1)\,V(\mathbf{r}_2)\,V(\mathbf{r}_3)\rangle = \gamma_3\,\delta(\mathbf{r}_1 - \mathbf{r}_3)\,\delta(\mathbf{r}_2 - \mathbf{r}_3)$. The constants γ_2 and γ_3 are parameters, characterizing the strength of the disorder potential and statistical properties of the random field. When the potential $V(\mathbf{r})$ is created by randomly distributed short-ranged impurities, we have $\gamma_2 = N_i\,v_0^2$ and $\gamma_3 = N_i\,v_0^3$, where N_i is the impurity concentration, and v_0 is the matrix element of the potential of an isolated impurity. In the case of Gaussian distribution, we have $\gamma_3 = 0$.

The calculation of the off-diagonal conductivity tensor as the loop Feynman diagram with the SO correction to the vertex part [8] gives us (side-jump mechanism of AHE)

$$\sigma_{xy}^{(sj)} = \frac{e^2\lambda_0^2}{6}(\nu_\downarrow\,k_{F\downarrow}\,v_{F\downarrow} - \nu_\uparrow\,k_{F\uparrow}\,v_{F\uparrow}), \qquad (2)$$

where $k_{F\uparrow,\downarrow}$, $v_{F\uparrow,\downarrow}$, and $\nu_{\uparrow,\downarrow}$ are the momenta, velocities, and the densites of states of majority and minority electrons at the Fermi surfaces, respectively.

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Now we consider the localization corrections to $\sigma_{xy}^{(sj)}$. They can be presented by the loop diagrams, which include Diffusons and Cooperons [9,10]. Assuming the exchange energy M larger than the inverse electron relaxation time, $1/\tau$, we may consider only triplet Cooperons and Diffusons, with the same orientation of spins in the particle–particle (Cooperon) or particle–hole (Diffuson) channels. There are eight loop diagrams for the correction to AHE containing the Cooperon and four diagrams with the Diffuson. We calculated the quantum corrections due to the Cooperon as a sum of all diagrams, and also due to the Diffuson, which renormalizes the anomalous vertex with the SO interaction. The calculation shows that the corrections related to the Cooperon, exactly cancel each other, whereas the contribution of the Diffuson gives a non-vanishing but very small in parameter $(\varepsilon_F\tau)^{-4}$ effect (ε_F is the Fermi energy). Thus, the localization corrections to $\sigma_{xy}^{(sj)}$ are negligibly small.

The localization corrections for the skew scattering have been calculated earlier [6] in a different model—without spin polarization of electron gas due to the Stoner-like itinerant field M but with a partial polarization of spin-orbit scatterers. To avoid possible differences related with the choice of model, we have calculated the localization corrections to the AHE due to the skew scattering from the Hamiltonian of Eq. (1). In frame of the skew scattering, we take into account the loop diagrams with the third-order corrections due to scattering from impurities, keeping the first order of SO-depending matrix elements. Without quantum corrections, the relevant diagrams for the skew scattering mechanism are two diagrams with such third-order corrections [8]. Calculating them and taking into account that $\langle V^2\rangle = \gamma_2/a_0^3$ and $\langle V^3\rangle = \gamma_3/a_0^6$ (here a_0 is the lattice parameter), we obtain

$$\sigma_{xy}^{(ss)} = \frac{\pi\langle V^3\rangle\,\lambda_0^2\,a_0^3}{6\langle V^2\rangle}\Big[\sigma_{xx,\downarrow}\,k_{F\downarrow}^2\,v_\downarrow - \sigma_{xx,\uparrow}\,k_{F\uparrow}^2\,v_\uparrow\Big], \tag{3}$$

where $\sigma_{xx,\uparrow,\downarrow}$ are the diagonal conductivities in the spin up and down channels. The dimensionless ratio $\langle V^3\rangle/\langle V^2\rangle^{3/2}$ depends only on the shape of the distribution $P(V)$ of the random field $V(\mathbf{r})$, whereas the combination $va_0^3\langle V^2\rangle^{1/2}$ characterizes the relative strength of the potential.

Now we consider the diagrams with one Cooperon and three-leg impurity vertices. There are twelve non-vanishing diagrams. After calculating them, we find for the skew scattering

$$\Delta\sigma_{xy}^{(ss)} = \frac{e^2\lambda_0^2\gamma_3}{8\sqrt{3}\,\pi\gamma_2}\left[\frac{k_{F\uparrow}^2\,v_\uparrow}{v_{F\uparrow}\,\tau_\uparrow^{1/2}}\left(\frac{1}{\tau_{0\uparrow}^{1/2}} - \sqrt{\frac{1}{\tau_{so\uparrow}} + \frac{1}{\tau_{\varphi\uparrow}}}\right) - \frac{k_{F\downarrow}^2\,v_\downarrow}{v_{F\downarrow}\,\tau_\downarrow^{1/2}}\left(\frac{1}{\tau_{0\downarrow}^{1/2}} - \sqrt{\frac{1}{\tau_{so\downarrow}} + \frac{1}{\tau_{\varphi\downarrow}}}\right)\right], \tag{4}$$

where $\tau_{so,\uparrow,\downarrow}$ and $\tau_{\varphi,\uparrow,\downarrow}$ are the spin–orbit and phase relaxation times [9,10], and $\tau_{0\uparrow,\downarrow}$ are some constants ($\tau_{0\uparrow,\downarrow}\simeq\tau_{\uparrow,\downarrow}$), which cannot be calculated exactly in the diffusion approximation for the Cooperon. In the effectively 2D case [9,10], similar calculations give us

$$\Delta\sigma_{xy}^{(ss)} = -\frac{e^2\lambda_0^2\gamma_3}{36\pi\gamma_2}\left[k_{F\uparrow}^2\,v_\uparrow\,\ln\left(\frac{\tau_\uparrow}{\tau_{so\uparrow}} + \frac{\tau_\uparrow}{\tau_{\varphi\uparrow}}\right) - k_{F\downarrow}^2\,v_\downarrow\,\ln\left(\frac{\tau_\downarrow}{\tau_{so\downarrow}} + \frac{\tau_\downarrow}{\tau_{\varphi\downarrow}}\right)\right]. \tag{5}$$

Thus, the localization correction to the AH conductivity due to the skew scattering is nonzero, in agreement with Ref. [6].

The anomalous Hall resistivity, determined as $R_{AH}\simeq\sigma_{xy}/\sigma_{xx}^2$, acquires the corrections from both diagonal and off-diagonal conductivities

$$\frac{\Delta R_{AH}}{R_{AH}^0} = \frac{\Delta\sigma_{xy}}{\sigma_{xy}^0} - 2\,\frac{\Delta\sigma_{xx}}{\sigma_{xx}^0}\,. \tag{6}$$

Since the correction to AH conductivity in frame of the side-jump mechanism is very small, the total localization correction $\Delta\sigma_{xy}$ is given by Eqs. (4) or (5). The relative magnitude of this correction depends on the prevailing mechanism of AH effect. Using Eqs. (2) and (3), we can find that the relative order of the AH conductivity due to the skew scattering or side-jump is

$$\frac{\sigma_{xy}^{(ss)}}{\sigma_{xy}^{(sj)}} \simeq \frac{v\,\gamma_3}{\gamma_2}(\varepsilon_F\tau). \tag{7}$$

The weak-localization approach is valid as long as $(\varepsilon_F\tau)\gg 1$. Thus, for $v\gamma_3/\gamma_2 > 1$, the skew scattering mechanism is more important, and the localization correction is determined by Eqs. (4) or (5). In the case of $v\gamma_3/\gamma_2 \ll 1$, the prevailing mechanism is side-jump. Since the side-jump correction is zero, the total localization correction, determined by Eqs. (4) or (5), turns out to be negligibly small: $\Delta\sigma_{xy}^{(ss)}/\sigma_{xy}^{(sj)} \simeq \left(\Delta\sigma_{xy}^{(ss)}/\sigma_{xy}^{(ss)}\right) \times\left[(v\gamma_3/\gamma_2)\,(\varepsilon_F\tau)\right]^{1/2} \ll \Delta\sigma_{xy}^{(ss)}/\sigma_{xy}^{(ss)}$.

Thus, (i) for the low-resistivity metals with prevailing skew scattering, the localization correction to AH resistivity Eq. (6) contains both parts with $\Delta\sigma_{xy}$ (described by Eqs. (4) or (5) and $\Delta\sigma_{xx}$. No cancellation between them is possible due to the separation of contributions from the different spin channels; (ii) for the high-resistivity metals or doped semiconductors with prevailing side-jump mechanism, the correction to $\Delta\sigma_{xy}$ is negligibly small, so that the localization correction to AH resistivity is exactly twice the relative correction to the diagonal conductivity (with the opposite sign).

These results differ significantly from what is known for the usual Hall effect, described by a Hall constant R_H. The localization correction to R_H is zero due to the mutual cancellation of contributions from the diagonal and off-diagonal conductivities [11,12]. On the other hand, considering the interaction corrections to R_H, it has been found that $\Delta\sigma_{xy}^{(\mathrm{int})} = 0$. Thus, the total quantum corrections to the Hall constant are reduced to $\Delta R_{\mathrm{H}}/R_{\mathrm{H}}^0 = -2\,(\Delta\sigma_{xx}^{(\mathrm{int})}/\sigma_{xx})$.

The experiments on amorphous Fe films [7] have shown that the quantum correction to the AH resistivity Eq. (6) is double the correction to the diagonal conductivity. This is in accordance with our result for the localization corrections under condition that the side-jump mechanism prevails. The latter is in agreement with the comparatively high resistivity of amorphous Fe films studied in Ref. [7]. Our main argument in favor of the prevailing side-jump mechanism [4] is that the random field experienced by the electrons in amorphous films is naturally described by a distribution $P\{V(\mathbf{r})\}$ with nearly equal probabilities of positive and negative deviations of the random potential $V(\mathbf{r})$ from zero. In such a case the parameter $v\gamma_3/\gamma_2$ in Eq. (7) is small since $\langle V^3\rangle/\langle V^2\rangle^{3/2} \ll 1$. The authors of the cited works [6,7] have given another explanation of the measurements: suppression of localization corrections to the off-diagonal conductivity due to very strong SO scattering ($\tau_{\mathrm{so}} \simeq \tau$), upon the prevailing skew scattering mechanism. Besides, the quantum corrections to the AH conductivity due to electron-electron interaction have been calculated for the skew scattering, and the cancellation of interaction corrections has been proved. It should be noted, however, that the Hartree diagrams were not taken into account in this calculation.

In conclusion, we have shown that the role of localization corrections is quite different for the skew scattering and side-jump mechanisms of AH effect. We suggest that the experimental results of Ref. [7] can be interpreted as a relative smallness of the localization correction to the off-diagonal conductivity upon the prevailing side-jump mechanism.

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