

Localization corrections to the anomalous Hall effect in a ferromagnet

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We calculate the localization corrections to the anomalous Hall conductivity related to the contribution of spin-orbit scattering into the current vertex (side-jump mechanism). We show that in contrast to the ordinary Hall effect, there exists a nonvanishing localization correction to the anomalous Hall resistivity. The correction to the anomalous Hall conductivity vanishes in the case of side-jump mechanism, but is nonzero for the skew scattering. The total correction to the off-diagonal conductivity related to both mechanisms, does not compensate the correction to the diagonal conductivity.

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I. INTRODUCTION

The anomalous Hall (AH) effect can be observed in magnetically ordered metals or semiconductors without external magnetic field.^{1,2} The key point of any explanation of this effect is the presence of spin-orbit (SO) interaction, which breaks the symmetry to spin rotations.

The theory of AH effect has been developed in numerous works.^{3–8} More recently, the interest in this effect has been growing^{9–14} due to the importance of the spin polarization and spin-orbit interaction for transport properties of materials and structures of spin electronics.^{15–18} In addition, the measurement of AH effect has been proven to be a useful tool to determine the magnitude of magnetization in structures with magnetic layers.¹⁸

Usually, two relevant mechanisms are distinguished—a skew scattering^{9,4} and a side-jump effect.^{20,21} It is commonly believed that the first mechanism prevails in low-resistivity metals, whereas the other one (side-jump) can be more significant for metal alloys or semiconductors with much larger resistivity.^{1,2}

The theory of localization corrections to the conductivity and Hall conductivity has been developed in details for nonmagnetic metals and heavily doped semiconductors,^{22–25} but not for magnetically ordered materials. In our recent works²⁶ we analyzed some effects related with localization and interaction corrections in ferromagnets and in multilayer structures with thin magnetic layers.

The role of quantum corrections (both localization and exchange interaction) to AH effect has been considered theoretically in Ref. 27 but only in the case of skew scattering. In the present work we consider the localization corrections in the framework of side-jump mechanism. Also, we revisit the calculation of localization corrections for the skew scattering in the model of itinerant magnetism and confirm the result²⁷ found in a model of impurities with ordered magnetic moments. We show that the results for the side-jump and skew scattering are quite different.

II. THE MODEL

We consider a ferromagnet with a strong exchange magnetization \mathbf{M} oriented along the z axis, and a SO relativistic term (we put $\hbar = 1$)

$$H = \int d^3r \psi^\dagger(\mathbf{r}) \left[-\frac{\nabla^2}{2m^*} - M\sigma_z - \frac{i\lambda_0^2}{4} [\boldsymbol{\sigma} \times \nabla V(\mathbf{r})] \cdot \nabla + V(\mathbf{r}) \right] \psi(\mathbf{r}), \quad (1)$$

where m^* is the electron effective mass, λ_0 is a constant, which measures the strength of the SO interaction, $V(\mathbf{r})$ is a random potential created by impurities or defects, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and $\psi^\dagger \equiv (\psi_\uparrow^\dagger, \psi_\downarrow^\dagger)$ is the spinor field, corresponding to electrons with spin up and down orientations. The constant λ_0 has the dimensionality of length. For nonrelativistic electrons in vacuum, λ_0 is equal to $\lambda_c/2\pi$, where $\lambda_c = 2\pi/m_0c$ is the Compton wavelength of electron and m_0 is the free electron mass.

We assume that the potential $V(\mathbf{r})$ is short ranged, with zero mean value $\langle V(\mathbf{r}) \rangle = 0$, where the angle brackets mean the configurational averaging over all realization of $V(\mathbf{r})$. We shall characterize this potential by its second γ_2 and third γ_3 momenta, denoting $\langle V(\mathbf{r}_1)V(\mathbf{r}_2) \rangle = \gamma_2 \delta(\mathbf{r}_1 - \mathbf{r}_2)$ and $\langle V(\mathbf{r}_1)V(\mathbf{r}_2)V(\mathbf{r}_3) \rangle = \gamma_3 \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_2 - \mathbf{r}_3)$.

It should be emphasized that the constants γ_2 and γ_3 are parameters, characterizing not only the strength of the disorder potential, but also the statistical properties of the random field. When the potential $V(\mathbf{r})$ is created by impurities, distributed randomly at some points \mathbf{R}_i , we have $V(\mathbf{r}) = \sum_i v(\mathbf{r} - \mathbf{R}_i)$. It results in $\gamma_2 = N_i v_0^2$ and $\gamma_3 = N_i v_0^3$, where N_i is the impurity concentration, and v_0 is the matrix element of the short-ranged potential of one isolated impurity, $v(\mathbf{r} - \mathbf{R}_i) = v_0 \delta(\mathbf{r} - \mathbf{R}_i)$. In the case of purely Gaussian potential, we should take $\gamma_3 = 0$.

Calculating the matrix elements of the Hamiltonian (1) in momentum representation, we obtain

$$H = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^\dagger \left(\frac{k^2}{2m^*} - M\sigma_z \right) \psi_{\mathbf{k}} + \sum_{\mathbf{k}\mathbf{k}'} \psi_{\mathbf{k}}^\dagger V_{\mathbf{k}-\mathbf{k}'} \left[1 + \frac{i\lambda_0^2}{4} (\mathbf{k} \times \mathbf{k}') \cdot \boldsymbol{\sigma} \right] \psi_{\mathbf{k}'}, \quad (2)$$

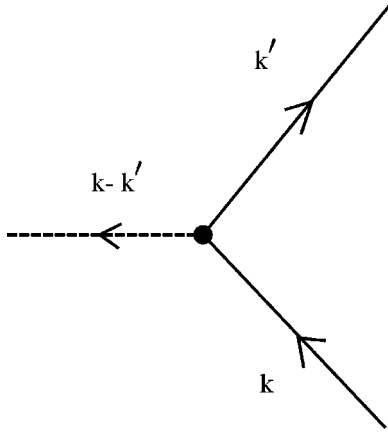


FIG. 1. Anomalous vertex for the coupling to electromagnetic field with SO interaction. The dashed line is for the impurity scattering and the filled black circle is for the external electromagnetic field.

where $V_{\mathbf{k}}$ is the Fourier transform of the potential $V(\mathbf{r})$. The second term in Eq. (2) describes the SO scattering from impurities.

To find the expression for current density operator $\mathbf{j}(t)$, we switch on an electromagnetic field $\mathbf{A}(t)$ in a gauge-invariant way, $\mathbf{k} \rightarrow (\mathbf{k} - e\mathbf{A}/c)$, and calculate the derivative

$$j_{\alpha} = -c \frac{\delta H}{\delta A_{\alpha}}, \quad (3)$$

which gives us

$$j_{\alpha} = \sum_{\mathbf{k}\mathbf{k}'} \psi_{\mathbf{k}}^{\dagger} \left[\frac{e}{m^*} \left(k_{\alpha} - \frac{eA_{\alpha}}{c} \right) \delta_{\mathbf{k}\mathbf{k}'} + \frac{ie\lambda_0^2}{4} V_{\mathbf{k}-\mathbf{k}'} \epsilon_{\alpha\beta\gamma} (k'_{\beta} - k_{\beta}) \sigma_{\gamma} \right] \psi_{\mathbf{k}'}, \quad (4)$$

where $\epsilon_{\alpha\beta\gamma}$ is the unit antisymmetric tensor.

According to Eq. (4), the SO interaction contributes to the current vertex in the Feynman diagrams of the conductivity tensor.²⁸ The additional anomalous vertex [second term in Eq. (4)] can be presented by a three-leg vertex, Fig. 1, where the dashed line corresponds to the interaction $V_{\mathbf{k}-\mathbf{k}'}$ with impurities, and the black point implies the coupling to the external electromagnetic field.

III. LOCALIZATION CORRECTIONS IN FRAME OF THE SIDE-JUMP MECHANISM

Calculating the Feynman diagrams for the off-diagonal (Hall) conductivity, Fig. 2, we find (an additional factor 2 comes from the contributions of right vertices)

$$\sigma_{xy}^{(sj)} = - \frac{ie^2\lambda_0^2\gamma_2}{4\pi m^*} \text{Tr} \sum_{\mathbf{k}\mathbf{k}'} \sigma_z k_y^2 G_{\mathbf{k}}^R G_{\mathbf{k}}^A (G_{\mathbf{k}'}^R - G_{\mathbf{k}'}^A), \quad (5)$$

where the retarded (R) and advanced (A) Green functions at the Fermi surface are diagonal matrices

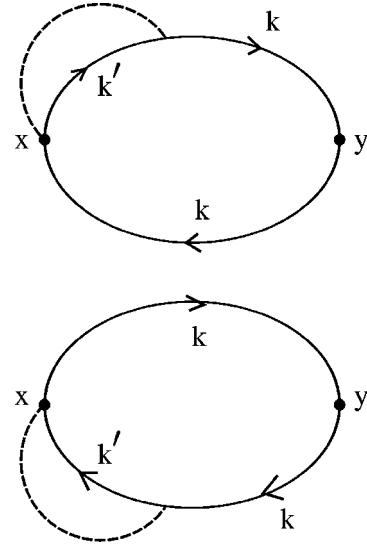


FIG. 2. Feynman diagrams for AH conductivity $\sigma_{xy}^{(sj)}$ (side-jump mechanism).

$$G_{\mathbf{k}}^{R,A} = \text{diag} \left(\frac{1}{\mu - \varepsilon_{\uparrow}(k) \pm i/(2\tau_{\uparrow})}, \frac{1}{\mu - \varepsilon_{\downarrow}(k) \pm i/(2\tau_{\downarrow})} \right). \quad (6)$$

Here $\varepsilon_{\uparrow,\downarrow}(k) = k^2/(2m^*) \mp M$ are the energy spectra of spin-up and spin-down electrons, respectively, μ is the chemical potential, and $\tau_{\uparrow,\downarrow}$ are the corresponding relaxation times. The relaxation times are determined by the scattering from the random potential, and they are equal to $\tau_{\uparrow,\downarrow} = (2\pi\nu_{\uparrow,\downarrow}\gamma_2)^{-1}$, where ν_{\uparrow} and ν_{\downarrow} are the densities of states for spin-up (majority) and spin-down (minority) electrons at the Fermi level.

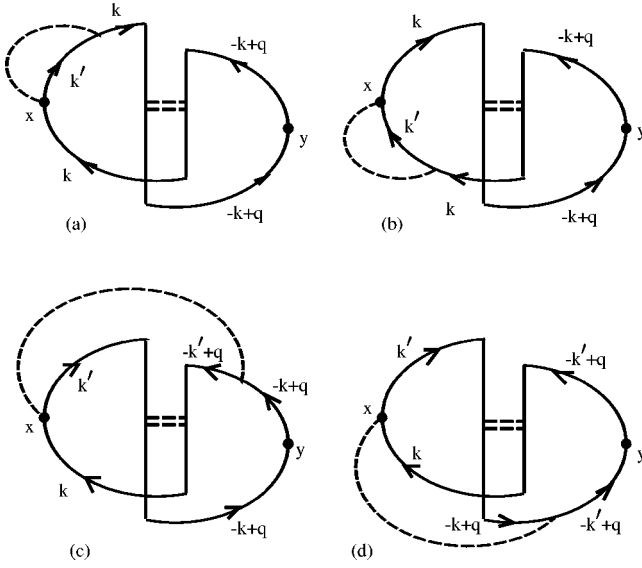
After calculating the integrals in Eq. (5), we find the side-jump AH conductivity¹⁴ (in final formulas we restore \hbar and use the electron parameters at the Fermi surfaces)

$$\sigma_{xy}^{(sj)} = \frac{e^2}{6\hbar} \lambda_0^2 (\nu_{\downarrow} \hbar k_{F\downarrow} v_{F\downarrow} - \nu_{\uparrow} \hbar k_{F\uparrow} v_{F\uparrow}), \quad (7)$$

where $k_{F\uparrow,\downarrow}$ and $v_{F\uparrow,\downarrow}$ are the momenta and velocities of majority and minority electrons at the Fermi surfaces, respectively.

Now we consider the localization corrections to $\sigma_{xy}^{(sj)}$. They can be presented by the loop diagrams with diffusons and Cooperons.²²⁻²⁴ Assuming the exchange energy M larger than $1/\tau$, we can restrict ourselves by considering only triplet Cooperons and diffusons, with the same orientation of spins in the particle-particle (Cooperon) or particle-hole (diffuson) channels. There are eight diagrams containing such Cooperons and four diagrams with Diffusons, presented in Figs. 3 and 4, respectively (the figures show only diagrams with left anomalous vertices).

We calculate first the quantum corrections due to the Cooperons. Calculating the first two diagrams of Fig. 3[(a) and (b)], we find


 FIG. 3. Localization corrections to $\sigma_{xy}^{(SJ)}$ due to the Cooperons.

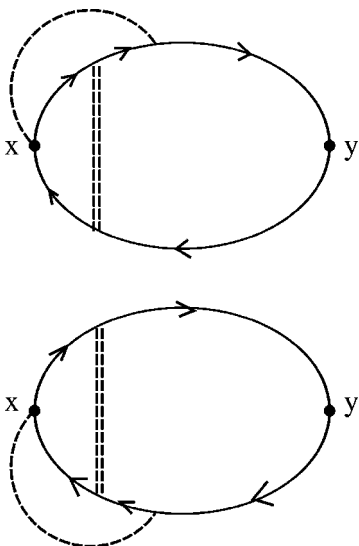
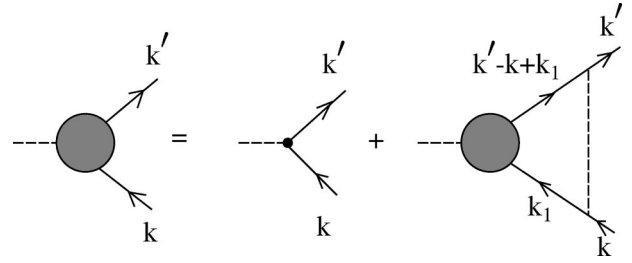
$$\Delta\sigma_{xy}^{(1)} = \frac{ie^2\lambda_0^2\gamma_2}{8\pi m^*} \text{Tr} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \sigma_z k_y^2 (G_{\mathbf{k}'}^R - G_{\mathbf{k}'}^A) \times (G_{\mathbf{k}}^R)^2 (G_{\mathbf{k}}^A)^2 C(0, \mathbf{q}), \quad (8)$$

where the spin components of the Cooperon²²⁻²⁴ are equal to

$$C_\sigma(\omega, \mathbf{q}) = \frac{1}{2\pi\nu_\sigma\tau_\sigma^2 - i\omega + D_\sigma q^2 + 1/\tau_{s\sigma, \sigma} + 1/\tau_{\varphi, \sigma}}. \quad (9)$$

Here $D_\sigma = v_{F\sigma}^2 \tau_\sigma / d$ is the diffusion constant of electrons (d the effective dimensionality²⁴), $\tau_{s\sigma, \sigma}$ and $\tau_{\varphi, \sigma}$ are the spin-orbit and phase relaxation times, respectively. In Eq. (8) we neglected small momentum $q \ll k \sim k_F$ in the arguments of Green's functions.

The calculation of two other diagrams of Fig. 3[(c) and (d)] gives us


 FIG. 4. Localization corrections to $\sigma_{xy}^{(sj)}$ due to the diffusons.

 FIG. 5. The equation for the renormalized anomalous vertex $\Gamma(\mathbf{k}, \mathbf{k}')$.

$$\Delta\sigma_{xy}^{(2)} = -\frac{ie^2\lambda_0^2\gamma_2}{8\pi m^*} \text{Tr} \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \sigma_z (k_y')^2 (G_{\mathbf{k}'}^R - G_{\mathbf{k}'}^A) G_{\mathbf{k}'}^R G_{\mathbf{k}'}^A G_{\mathbf{k}}^R G_{\mathbf{k}}^A C(0, \mathbf{q}). \quad (10)$$

After integrating over \mathbf{k} and \mathbf{k}' , we find that the contributions of all diagrams with Cooperons, Eqs. (8) and (10), cancel each other exactly. This result can also be seen by comparing directly Eqs. (8) and (10) and by using the property of Green's functions $G_{\mathbf{k}}^R - G_{\mathbf{k}}^A = (-i/\tau) G_{\mathbf{k}}^R G_{\mathbf{k}}^A$.

The diagrams with diffusons (Fig. 4) can be taken into account as a renormalization of the anomalous vertex by impurities. As is known, the normal electromagnetic vertex without SO correction [first term in Eq. (4)] can be renormalized only for nonpointlike defect.²⁸ Here we show that the anomalous vertex is renormalized in the case pointlike defects too.

The equation for the three-leg vertex has the following form (Fig. 5):

$$\Gamma(\mathbf{k}, \mathbf{k}') = \Gamma^0(\mathbf{k}, \mathbf{k}') + \gamma_2 \sum_{\mathbf{k}_1} G_{\mathbf{k}_1}^A \Gamma(\mathbf{k}_1, \mathbf{k}_1) + (\mathbf{k}' - \mathbf{k}) G_{\mathbf{k}' - \mathbf{k} + \mathbf{k}_1}^R, \quad (11)$$

where

$$\Gamma_\alpha^0(\mathbf{k}, \mathbf{k}') = \frac{ie\lambda_0^2}{4} V_{\mathbf{k}-\mathbf{k}'} \epsilon_{\alpha\beta\gamma} (k'_\beta - k_\beta) \sigma_\gamma. \quad (12)$$

In view of Eq. (12), we can look for a solution of Eq. (11) in the form of matrix in the spin space that depends only on the difference of momenta, $\Gamma_\alpha(\mathbf{k}, \mathbf{k}') = \Gamma_\alpha(\mathbf{k}' - \mathbf{k})$. Hence, the solution can be presented as

$$\Gamma_\alpha(\mathbf{q}) = \Gamma_\alpha^0(\mathbf{q}) [1 - \gamma_2 \Pi(\mathbf{q})]^{-1}, \quad (13)$$

where $\Pi(\mathbf{q})$ is the diagonal matrix

$$\Pi(\mathbf{q}) = \sum_{\mathbf{k}} G_{\mathbf{k}+\mathbf{q}}^R G_{\mathbf{k}}^A. \quad (14)$$

According to Eqs. (11) and (12), we can present the x component of the vertex $\Gamma(\mathbf{q})$ as

$$\Gamma_x(\mathbf{q}) = q_y \Gamma_x^y(\mathbf{q}) + q_z \Gamma_x^z(\mathbf{q}), \quad (15)$$

where

$$\Gamma_x^y(\mathbf{q}) = \frac{ie\lambda_0^2}{4} V_{\mathbf{q}} \text{diag} \left(\frac{1}{D_{\uparrow} q^2 \tau_{\uparrow}}, -\frac{1}{D_{\downarrow} q^2 \tau_{\downarrow}} \right), \quad (16)$$

and only the $\Gamma_x^y(\mathbf{q})$ component is needed, since the second term in Eq. (15) gives the vanishing contribution to $\Delta\sigma_{xy}^{(sj)}$.

Using Eqs. (15) and (16), we find the localization corrections to $\sigma_{xy}^{(sj)}$

$$\Delta\sigma_{xy}^{(sj)} = \frac{ie^2\lambda_0^2\gamma_2}{8\pi m^*} \text{Tr} \sum_{\mathbf{k}\mathbf{q}} \sigma_z \frac{q_y k_y}{D_{\sigma} q^2 \tau_{\sigma}} (G_{\mathbf{k}+\mathbf{q}}^R + G_{\mathbf{k}-\mathbf{q}}^A) G_{\mathbf{k}}^R G_{\mathbf{k}}^A. \quad (17)$$

Taking into account that very small momenta q can be essential for the diffuson, $q < (D\tau)^{-1/2} \ll k \sim k_F$, we can present Eq. (17) in the form

$$\Delta\sigma_{xy}^{(sj)} = -\frac{ie^2\lambda_0^2\gamma_2}{8\pi m^*} \text{Tr} \sum_{\mathbf{k}\mathbf{q}} \sigma_z \frac{q_y^2}{D_{\sigma} q^2 \tau_{\sigma}} \times (G_{\mathbf{k}}^R - G_{\mathbf{k}}^A) G_{\mathbf{k}}^R G_{\mathbf{k}}^A, \quad (18)$$

and, after calculating the integral over \mathbf{k} , we find

$$\Delta\sigma_{xy}^{(sj)} = -\frac{e^2\lambda_0^2}{4\pi m^*} \text{Tr} \sum_{\mathbf{q}} \sigma_z P(\mathbf{q}), \quad (19)$$

where $P(\mathbf{q})$ is a diagonal matrix with the elements

$$P_{\sigma}(\mathbf{q}) \simeq \frac{q_y^2}{D_{\sigma} q^2}. \quad (20)$$

The integral over q in Eq. (19) is mainly determined by the diffuson at the upper limit, $q \sim (D\tau)^{-1/2}$, for which the vertex $\Gamma_x^y(\mathbf{q})$, Eq. (16), was not found correctly.

In the three-dimensional case we can estimate the integral as

$$\int \frac{d^3q}{(2\pi)^3} P_{\sigma}(\mathbf{q}) \simeq \frac{m^* k_{F\sigma}^3}{(\varepsilon_{F\sigma} \tau_{\sigma})^4}. \quad (21)$$

Combining Eqs. (19),(20) with Eq. (6), we find the relative value of the quantum correction $\Delta\sigma_{xy}^{(sj)}/\sigma_{xy}^{(sj)} \simeq (\varepsilon_F \tau)^{-4}$. Since the usual correction to the conductivity σ_{xx} has the relative magnitude of $(\varepsilon_F \tau)^{-2}$, the localization correction to the off-diagonal conductivity $\sigma_{xy}^{(sj)}$ turns out to be very small.

In the case of effective two dimensionality of quantum corrections (when the thickness of magnetic film d obeys inequalities $d \ll L_{\varphi}, L_{SO}$, where L_{φ} is the phase-breaking length and L_{SO} is the SO scattering length), using Eq. (19), we get $\Delta\sigma_{xy}^{(sj)}/\sigma_{xy}^{(sj)} \sim (\varepsilon_F \tau)^{-3}$, whereas $\Delta\sigma_{xx}/\sigma_{xx} \sim (\varepsilon_F \tau)^{-1}$. Thus, the correction to $\sigma_{xy}^{(sj)}$ can be neglected for any effective dimensionality.

IV. LOCALIZATION CORRECTIONS IN CASE OF THE SKEW SCATTERING

The localization corrections for the skew scattering has been calculated earlier.²⁷ It should be noted, however, that the result of Ref. 27 was obtained in a different model—

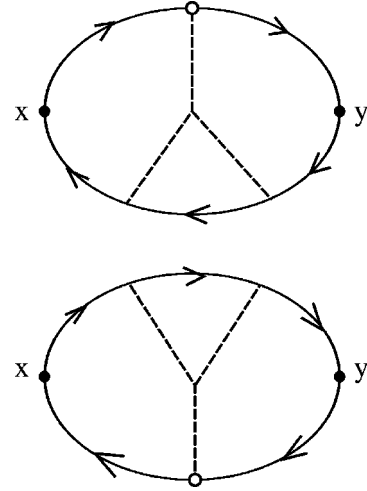


FIG. 6. Diagrams for $\sigma_{xy}^{(SS)}$ (skew scattering mechanism). The impurity scattering is in the third order, and the spin-orbit scattering amplitude is denoted by unfilled circle.

without spin polarization of electron gas due to the Stoner-like itinerant field M [the second term in the Hamiltonian (1)] but with a partial polarization of spin-orbit scatterers. To avoid possible differences related with the choice of model, we have calculated the localization corrections to the AH effect due to the skew scattering from the Hamiltonian of Eq. (1).

In frame of the skew scattering, we take into account the diagrams with the third-order corrections due to scattering from impurities, keeping the first order of SO-depending matrix elements. Without quantum corrections, the relevant diagrams for the skew scattering mechanism are presented in Fig. 6.¹⁴ Calculating these diagrams, we find

$$\sigma_{xy}^{(SS)} = \frac{\pi e^2 \lambda_0^2}{18\hbar} \left(k_{F\downarrow}^2 \nu_{\downarrow} v_{F\downarrow}^2 \tau_{\downarrow} \frac{\nu_{\downarrow} \gamma_3}{\gamma_2} - k_{F\uparrow}^2 \nu_{\uparrow} v_{F\uparrow}^2 \tau_{\uparrow} \frac{\nu_{\uparrow} \gamma_3}{\gamma_2} \right). \quad (22)$$

In this formula, the dimensionless factor $\nu\gamma_3/\gamma_2$ contains the information about both the strength of the random potential and its statistical properties. To make it more physically sound, we introduce the average values of the second and third powers of the mean potential at an elementary cell $\langle V^2 \rangle$ and $\langle V^3 \rangle$.

Taking into account that $\langle V^2 \rangle = \gamma_2/a_0^3$ and $\langle V^3 \rangle = \gamma_3/a_0^6$ (where a_0 is the lattice parameter), we obtain

$$\sigma_{xy}^{(SS)} = \frac{\pi}{6} \frac{\langle V^3 \rangle}{\langle V^2 \rangle^{3/2}} [\sigma_{xx,\downarrow} (\lambda_0 k_{F\downarrow})^2 (\nu_{\downarrow} a_0^3 \langle V^2 \rangle^{1/2}) - \sigma_{xx,\uparrow} (\lambda_0 k_{F\uparrow})^2 (\nu_{\uparrow} a_0^3 \langle V^2 \rangle^{1/2})]. \quad (23)$$

Here the dimensionless factor $\langle V^3 \rangle / \langle V^2 \rangle^{3/2}$ depends only on statistical properties of the random field $V(\mathbf{r})$, whereas the dimensionless combination $\nu a_0^3 \langle V^2 \rangle^{1/2}$ characterizes the relative strength of the potential.

Now we consider the diagrams with one Cooperon and three-leg impurity vertices. There are twelve nonvanishing diagrams of the type similar to that presented in Fig. 7. Here

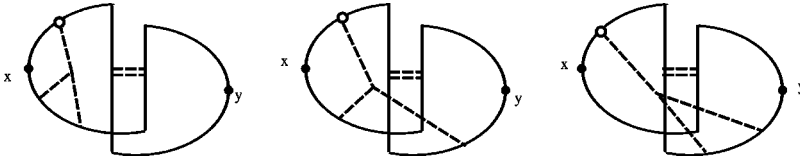


FIG. 7. First three diagrams describing localization corrections to $\sigma_{xy}^{(SS)}$. Other diagrams differ by locations of the spin-orbit vertex (unfilled circle).

the SO-dependent vertex (indicated by the white circle) lies on one of four possible Green function lines. The other nine diagrams are similar to those of Fig. 7, but differ by the location of the SO vertex. The corrections from diagrams with diffusons vanish.

After calculating all the diagrams, we find for the skew scattering

$$\Delta\sigma_{xy}^{(SS)} = \frac{\pi e^2 \gamma_3}{9 \gamma_2} \lambda_0^2 \left(k_{F\uparrow}^2 \nu_{\uparrow}^2 \nu_{F\uparrow}^2 \tau_{\uparrow}^3 \sum_{\mathbf{q}} C_{\uparrow}(0, \mathbf{q}) - k_{F\downarrow}^2 \nu_{\downarrow}^2 \nu_{F\downarrow}^2 \tau_{\downarrow}^3 \sum_{\mathbf{q}} C_{\downarrow}(0, \mathbf{q}) \right). \quad (24)$$

Using Eq. (9) and calculating the integral over \mathbf{q} , the skew-scattering correction can be presented in the three-dimensional case as

$$\Delta\sigma_{xy}^{(SS)} = \frac{e^2}{8\sqrt{3}\pi\hbar} \lambda_0^2 \left\{ k_{F\uparrow}^2 \frac{\nu_{\uparrow} \gamma_3}{\gamma_2} \frac{1}{\nu_{F\uparrow} \tau_{\uparrow}^{1/2}} \times \left[\frac{1}{\tau_{0\uparrow}^{1/2}} - \left(\frac{1}{\tau_{so\uparrow}} + \frac{1}{\tau_{\varphi\uparrow}} \right)^{1/2} \right] - k_{F\downarrow}^2 \frac{\nu_{\downarrow} \gamma_3}{\gamma_2} \frac{1}{\nu_{F\downarrow} \tau_{\downarrow}^{1/2}} \times \left[\frac{1}{\tau_{0\downarrow}^{1/2}} - \left(\frac{1}{\tau_{so\downarrow}} + \frac{1}{\tau_{\varphi\downarrow}} \right)^{1/2} \right] \right\}, \quad (25)$$

where $\tau_{0\sigma}$ are some constants ($\tau_{0\sigma} \approx \tau_{\sigma}$), which cannot be calculated exactly in the diffusion approximation of Eq. (9).

In the effectively two-dimensional case, similar calculations give us

$$\Delta\sigma_{xy}^{(SS)} = -\frac{e^2}{36\pi\hbar} \lambda_0^2 \left\{ k_{F\uparrow}^2 \frac{\nu_{\uparrow} \gamma_3}{\gamma_2} \ln \left[\tau_{\uparrow} \left(\frac{1}{\tau_{so\uparrow}} + \frac{1}{\tau_{\varphi\uparrow}} \right) \right] - k_{F\downarrow}^2 \frac{\nu_{\downarrow} \gamma_3}{\gamma_2} \ln \left[\tau_{\downarrow} \left(\frac{1}{\tau_{so\downarrow}} + \frac{1}{\tau_{\varphi\downarrow}} \right) \right] \right\}. \quad (26)$$

Thus, the localization correction to the AH conductivity due to the skew scattering is nonzero, in agreement with Ref. 27.

V. DISCUSSION AND CONCLUSION

The anomalous Hall resistivity, determined as

$$R_{AH} = \frac{\sigma_{xy}}{\sigma_{xx}^2}, \quad (27)$$

acquires the corrections from both diagonal and off-diagonal conductivities

$$\frac{\Delta R_{AH}}{R_{AH}^0} = \frac{\Delta\sigma_{xy}}{\sigma_{xy}^0} - 2 \frac{\Delta\sigma_{xx}}{\sigma_{xx}^0}. \quad (28)$$

Since the correction to AH conductivity in frame of the side-jump mechanism is zero, the total localization correction $\Delta\sigma_{xy}$ is given by Eqs. (25) or (26). The relative magnitude of this correction depends on the prevailing mechanism of AH effect. Using Eqs. (7) and (22), we can find that the relative order of the AH conductivity due to the skew scattering or side-jump is

$$\frac{\sigma_{xy}^{(SS)}}{\sigma_{xy}^{(SJ)}} \approx \frac{\nu \gamma_3}{\gamma_2} (\varepsilon_F \tau). \quad (29)$$

The weak-localization approach is valid as long as $(\varepsilon_F \tau) \gg 1$. Thus, for $\nu \gamma_3 / \gamma_2 > 1$, the skew scattering mechanism is more important, and the localization correction is determined by Eqs. (25) or (26). In the case of $\nu \gamma_3 / \gamma_2 \ll 1$, the prevailing mechanism is side-jump. Since the side-jump correction is zero, the total localization correction, determined by Eqs. (25) or (26), turns out to be negligibly small: $(\Delta\sigma_{xy}^{(SS)} / \sigma_{xy}^{(SJ)}) \approx (\Delta\sigma_{xy}^{(SS)} / \sigma_{xy}^{(SS)}) [(\nu \gamma_3 / \gamma_2) (\varepsilon_F \tau)]^{1/2} \ll (\Delta\sigma_{xy}^{(SS)} / \sigma_{xy}^{(SS)})$.

Collecting all together, we can formulate our final result as follows.

(i) For the low-resistivity metals with prevailing skew scattering, the localization correction to AH resistivity (28) contains both parts with $\Delta\sigma_{xy}$ [described by Eq. (25) or (26)] and $\Delta\sigma_{xx}$. No cancellation between them is possible due to the separation of contributions from the different spin channels.

(ii) For the high-resistivity metals or doped semiconductors with prevailing side-jump mechanism, the correction to $\Delta\sigma_{xy}$ is negligibly small, so that the localization correction to AH resistivity, Eq. (28), is exactly twice the relative correction to the diagonal conductivity (with the opposite sign).

These results, concerning the AH effect, differ significantly from what is known for the usual Hall effect, described by a Hall constant R_H . It has been shown²⁹⁻³¹ that the localization correction to R_H , determined by an analogous formula (28), is identically zero due to the mutual cancellation of contributions from the diagonal $\Delta\sigma_{xx}^{(loc)}$ and off-diagonal $\Delta\sigma_{xy}^{(loc)}$, conductivities. On the other hand, considering the interaction corrections to R_H , it has been found that $\Delta\sigma_{xy}^{(int)} = 0$. Thus, the total quantum corrections to the Hall constant are reduced to $\Delta R_H / R_H^0 = -2 (\Delta\sigma_{xx}^{(int)} / \sigma_{xx})$.

The experiments on amorphous ferromagnetic Fe films³² have shown that the quantum correction to the AH resistivity (28) is double the correction to the diagonal conductivity. This is in accordance with our result for the localization cor-

rections under condition that the side-jump mechanism prevails. The latter is in agreement with the comparatively high resistivity of amorphous Fe films studied in Ref. 32.

But our main argument in favor of the prevailing side-jump mechanism²⁰ is that the random field experienced by the electrons in amorphous films is more naturally described by a distribution $P\{V(\mathbf{r})\}$ with nearly equal probabilities of positive and negative deviations of the random potential $V(\mathbf{r})$ from zero. In such a case the parameter $\nu\gamma_3/\gamma_2$ in Eq. (29) is small thanks to $\langle V^3 \rangle / \langle V^2 \rangle^{3/2} \ll 1$.

The authors of Refs. 27,32 have given another explanation of the measurements: suppression of localization corrections to the off-diagonal conductivity due to very strong spin-orbit scattering ($\tau_{so} \approx \tau$), upon the prevailing skew scattering mechanism. In addition, the quantum corrections to the AH conductivity due to electron-electron interaction have been also calculated for the skew scattering in Ref. 27, and

the cancellation of interaction corrections in $\Delta\sigma_{xy}^{(SS, int)}$ has been proved. It should be noted, however, that the Hartree diagrams were not taken into account in this calculation.

In conclusion, we have shown that the role of localization corrections is quite different for the skew scattering and side-jump mechanisms of AH effect. We suggest that the experimental results of Ref. 32 can be interpreted as a relative smallness of the localization correction to the off-diagonal conductivity upon the prevailing side-jump mechanism.

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