

## Entanglement between nitrogen vacancy spins in diamond controlled by a nanomechanical resonator

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(Received 8 January 2013; published 7 August 2013)

One of the main challenges in spin qubits' studies associated with nitrogen vacancy impurities in diamond is to increase the coupling strength between the spins. With this task in mind we suggest a new type of a hybrid magneto-nano-electromechanical resonator, the functionality of which is based on a magnetic-field-induced deflection of an appropriate cantilever that oscillates between nitrogen vacancy (NV) spins in diamond. Specifically, we consider a Si(100) cantilever coated with a thin magnetic Ni film. As a new aspect of this study we utilize magnetoelastic stress and magnetic-field-induced torque to induce a controlled cantilever deflection. It is shown that, depending on the value of the system parameters, the induced asymmetry of the cantilever deflection substantially modifies the characteristics of the system. In particular, the coupling strength between the NV spins and the degree of entanglement can be controlled through magnetoelastic stress and magnetic-field-induced torque effects. Our theoretical proposal can be implemented experimentally with the potential of increasing several times the coupling strength between the NV spins. It finds that the coupling strength achieved by using our proposal enhances several times the maximal coupling strength reported before by Rabl *et al.* [P. Rabl, P. Cappellaro, M. V. Gurudev Dutt, L. Jiang, J. R. Maze, and M. D. Lukin, *Phys. Rev. B* **79**, 041302(R) (2009)].

DOI: [10.1103/PhysRevB.88.085201](https://doi.org/10.1103/PhysRevB.88.085201)

PACS number(s): 85.85.+j, 03.67.Bg, 71.55.-i, 07.10.Cm

### I. INTRODUCTION

Nanoelectromechanical resonators (NAMRs) are attracting intense research efforts due to a number of favorable properties such as the high sensitivity and the swift response to an external force with a low power consumption. This makes NAMRs attractive for applications, e.g., for microwave switches, nanomechanical memory elements, and for single molecule sensing.<sup>1-3</sup> The role of quantized mechanical motion coupled to other quantum degrees of freedom as well as the influence of dissipation and noise are important issues in NAMRs research with numerous findings and demonstrations of applications, see, e.g., Refs. 4–24 and further references therein.

These studies also evidence the potential of NAMRs for studying fundamental questions concerning the quantum-classical interrelation and issues related to entanglement and quantum correlations. Particularly interesting for the present work are spin states in nitrogen vacancy (NV) impurities in diamond<sup>25-32</sup> as utilized for quantum information studies. Their quantum mechanical properties can be mapped onto effective two-level systems which possess very long decoherence times even at room temperature. On the other hand, they allow for a high degree of tunability via external magnetic fields which renders possible the use of NV impurity spins as sensors. For instance, a detection of a single electronic spin by a classical cantilever was demonstrated in Ref. 28. Magnetic tips attached on the free end of a cantilever generate a magnetic field gradient during oscillations, which induces a magnetic coupling between the nanomechanical oscillator and the spin system. Using the backaction due to this coupling one can readout the cantilever motion.<sup>25,26</sup>

One further fascinating application for the NV spin-based nanoelectromechanical resonator system is magnetic resonance force microscopy (MRFM) which was proposed to improve the detection resolution for the three-dimensional imaging of macromolecules.<sup>28</sup> The operation of the MRFM is based on the detection of the magnetic force between a ferromagnetic tip and spins. The possibility to achieve strong (up to 0.1MHz or even stronger) coupling between the quantized motion of the nanoresonator and NV impurity spin was demonstrated in Ref. 30. However, an efficient control of the coupling strength and the development of ultrasensitive cantilever-based force sensors is still a fundamental challenge. Another idea aims at using such a resonator to generate controllable entanglement between spins. To our knowledge, despite the rather long decoherence time, this proposal has not been demonstrated yet.

In this paper we propose a new type of a nanoelectromechanical resonator system, the functionality of which is based on the NV spin controlled by a magnetic field. The nanoresonator is a cantilever, which is covered with a magnetic film, such that we can exploit magnetoelastic stress or magnetic torque effects for a controllable cantilever deflection. At the free end of the cantilever magnetic tips are mounted, as indicated in Fig. 1. The magnetic field allows for a full control of the cantilever configuration via the amplitude of the applied magnetic field. This is the major novelty of the current proposal. As will be demonstrated below, the field-induced deflection of the cantilever has a major impact on the interaction strength between the NV spins and the magnetic tips, as well as on the strength of the indirect interaction between the spins mediated by the cantilever. It should be

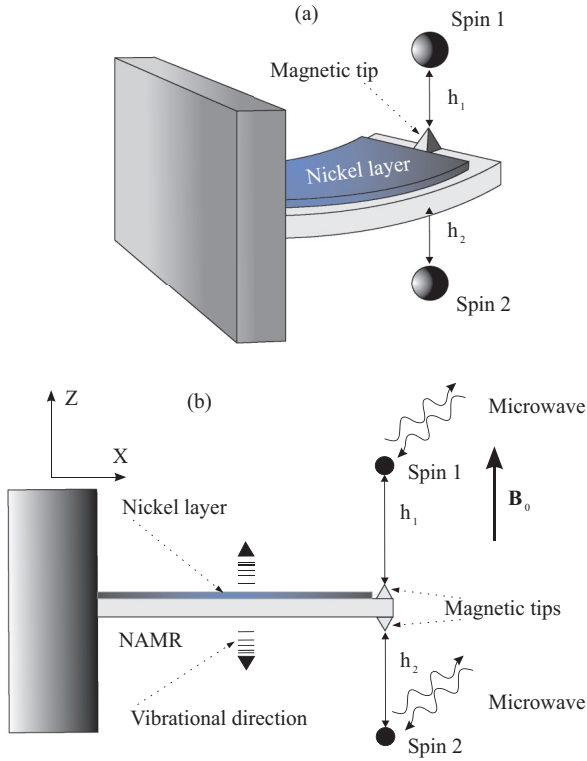


FIG. 1. (Color online) Schematics of the nanomechanical resonator system discussed in the present project. (a) The distances  $h_{1,2}$  between the NV spins and magnetic tips can be adjusted by the field-induced deflection of the cantilever. The deflection is controlled by the applied magnetic field  $B_0$ . In the absence of the external constant magnetic field  $B_0 = 0$  the system becomes symmetric, and  $h_1 = h_2 = h_0 \approx 25$  nm (b). Microwaves are used to drive spin transitions at the NV centers of spin 1 and spin 2. (b) Two magnetic tips are mounted on the free end of the cantilever, which mechanically oscillates in the  $z$ -axis direction. The length of the cantilever is of the order  $L \sim 3000$  nm. The cantilever surface is covered with a 10-nm thin Ni film. See Sec. III for further details.

noted that the increasing of the coupling strength between NV spins is highly desirable for realizing spin qubits based on NV impurities in diamond. We will demonstrate that using our proposal the coupling strength between NV spins can be increased at least three times as compared to the typical maximal coupling strength that has been reported before.<sup>25</sup> We will show that the asymmetry of the nanomechanical system with respect to the shape of the cantilever controlled by magnetic fields changes the degree of entanglement as well.

The paper is organized as follows. In the next section we introduce our system as well as the model Hamiltonian and discuss the relevant experimental parameters. Section III describes the magnetic-field-induced cantilever deflection. The subsequent Sec. IV focuses on the effects of the induced indirect interaction of the impurities mediated by the cantilever coupling. The influence of the asymmetry coupling on the degree of entanglement is considered in Sec. V. Finally, Sec. VI summarizes the results and provides some perspectives for further progress.

## II. THEORETICAL MODEL

The system of our interest is schematically shown in Fig. 1. Magnetic tips are attached to the free ends of the cantilever on both sides. The deflection of the cantilever and consequently the distance between magnetic tips and spins  $h_{1,2}$  is controlled via the magnetic field  $\vec{B}_0$  applied along the  $Z$  axis. The nitrogen vacancy center in diamond consists of a substitutional nitrogen atom with an adjacent vacancy. The total spin of the many-electron orbital ground state of the NV center is described by the spin triplet  $S = 1$ ,  $m_S = -1, 0, 1$ . NV spin is quantized along the N-V symmetry axis  $\langle 111 \rangle$  directed perpendicular to the main diagonal of the crystal.<sup>32</sup> In our case, the  $Z$  axis and external magnetic field also are directed parallel to the  $\langle 111 \rangle$  axis [see Fig. 1 (b)]. States with different  $|m_S\rangle$  are separated by a zero field splitting barrier<sup>25</sup> which is of the order  $\omega_0 = 2.88$  GHz. This splitting is an intrinsic property of the NV spin system and originates<sup>32</sup> from the effect of spin-spin interactions leading to the single-axis spin anisotropy  $DS_z^2 \approx \hbar\omega_0$ . In the theoretical description we set  $\hbar = 1$ . The role of the applied external magnetic field is twofold: First of all, the applied external magnetic field  $\mu_B B_0 < \hbar\omega_0$ , due to the Zeeman shift proportional to  $B_0 S_z$ , removes the degeneracy of the levels  $|-1\rangle, |1\rangle$ . Besides, as will be shown below, the external magnetic field, due to the thin magnetic Ni film deposited on the cantilever, modifies the shape of the cantilever. We will demonstrate that the asymmetry of the cantilever position between spins 1 and 2 (Fig. 1), induced by an external magnetic field, has important consequences for the coupling strength and the entanglement between the NV spins. Therefore, one can control the degree of entanglement and increase the interaction strength between the NV spins. This constitutes an important advance in nanomechanics.

The Hamiltonian of the single NV spin system reads<sup>25</sup>

$$H_{\text{NV}} = \sum_{i=\pm 1} \left( -\delta_i |i\rangle\langle i| + \frac{\Omega_i}{2} (|0\rangle\langle i| + |i\rangle\langle 0|) \right). \quad (1)$$

In the case of a weak magnetic field  $\mu_B B_0 \ll \hbar\omega_0$  ( $B_0 \ll 30$  mT) one can neglect level splitting and set  $\delta_- = \delta_+$ . In this case Rabi frequency  $\Omega_-$  for the transition between levels  $|0\rangle, |-1\rangle$  is equal to the transition frequency  $\Omega_+$  between levels  $|0\rangle, |1\rangle$ . The Hamiltonian (1) couples the ground state  $|0\rangle$  to the “bright” superposition of the excited states  $|b\rangle = \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$ , while the “dark” state  $|d\rangle = \frac{1}{\sqrt{2}}(|-1\rangle - |1\rangle)$  is readout and decoupled from the process.<sup>25</sup> Since only two states are involved, the NV spin triplet can be described via a  $S = 1/2$  pseudospin model. If the external constant magnetic field is strong enough, the splitting between the levels  $|-1\rangle, |1\rangle$  is larger and the system becomes identical to the three-level generalized Jaynes-Cummings model in the so-called lambda configuration<sup>33,34</sup> (see Fig. 2).

A peculiarity of the generalized Jaynes-Cummings model is the expectation of two different transition frequencies between states  $(|0\rangle, |-1\rangle)$  and  $(|0\rangle, |1\rangle)$ . The Hamiltonian of the system holds a  $SU(3)$  symmetry and can be specified in terms of the Gell-Mann generators.<sup>33</sup> The three-level Jaynes-Cummings model is exactly solvable in the general case. If the rf field contains only one frequency resonant to the transition  $(|0\rangle, |-1\rangle)$  the system can be reduced to the

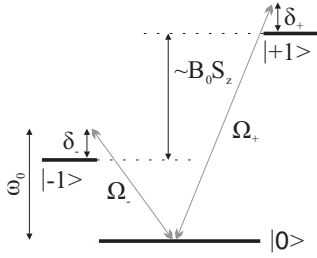


FIG. 2. Configuration of the energy levels.  $\Omega_{\pm}$  stand for the Rabi frequencies between the ground and the excited levels ( $|0\rangle, |-1\rangle$ ) and ( $|0\rangle, |1\rangle$ ).  $\delta_{\pm}$  denote the detuning between microwave frequency  $\omega_0$  and transition frequencies.

$S = 1/2$  pseudospin model which will describe transitions between the states  $|-1\rangle, |0\rangle$  only since the transition between the levels  $|1\rangle, |0\rangle$  is off resonance and therefore forbidden. However, the Rabi frequency  $\Omega_-$  for the transition between the states  $|-1\rangle, |0\rangle$  in this case is different from the Rabi frequency corresponding to the transition between the ground state  $|0\rangle$  and the bright superposition of the excited states  $|b\rangle = \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle)$ . Nevertheless, in both cases (that means transition  $|0\rangle \rightarrow |b\rangle$  for the zero field case  $B = 0$  and transition  $|0\rangle \rightarrow |-1\rangle$  for the finite field case  $B \neq 0$ ) our system can be described via the effective two-level model with a different Rabi transition frequency. The Hamiltonian of the system in the frame rotating with the frequency of the rf field has the form [see Eq. (1) in Ref. 26]

$$H_S = -\delta|-1\rangle\langle-1| + \frac{\Omega(B)}{2}(|-1\rangle\langle 0| + |0\rangle\langle-1|). \quad (2)$$

Here  $\delta$  is the detuning between the microwave frequency and the intrinsic frequency of the spins. The Rabi frequency of the transition between up and down spin states  $\Omega(B)$  depends on the amplitude of the magnetic field<sup>25</sup> and, thus, includes both limiting cases. In the case of zero external constant magnetic field  $B = 0$  and zero splitting between the levels  $|-1\rangle, |1\rangle$  the Rabi frequency for the transition between the bright  $|b\rangle$  and the ground  $|0\rangle$  states is equal to  $\Omega(0) = \Omega_0$ , while in the case of a nonzero external field and nonzero splitting for the transition between levels  $|-1\rangle, |0\rangle$  the Rabi frequency is equal to

$$\Omega(B) = \Omega_0 - \Delta\Omega(B), \quad \Delta\Omega(B) = \mu_B(B_0 + B_{ms}). \quad (3)$$

Here  $B_0$  is the amplitude of the external constant magnetic field applied on the system,  $B_{ms}$  is the magnetic field produced by the magnetic film on the cantilever at the position of the spins 1 and 2. The eigenbasis of the Hamiltonian (2) is given by the following states<sup>26</sup>

$$\begin{aligned} |g\rangle &= \cos(\theta/2)|-1\rangle + \sin(\theta/2)|0\rangle, \\ |e\rangle &= -\sin(\theta/2)|-1\rangle + \cos(\theta/2)|0\rangle, \\ \tan\theta &= -\frac{\Omega}{\delta}. \end{aligned} \quad (4)$$

In the basis (4) the components of the pseudospin operator have the form

$$\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \quad \sigma_+ = |e\rangle\langle g|, \quad \sigma_- = |g\rangle\langle e| \quad (5)$$

while the  $Z$  component of the spin  $S_z = \frac{1}{2}(|0\rangle\langle 0| - |-1\rangle\langle-1|)$  and the Hamiltonian of the system  $H_S$  [given by Eq. (2)] reads

$$\begin{aligned} S_z &= \frac{1}{2}[\cos\theta\sigma_z + \sin\theta(\sigma_+ + \sigma_-)], \\ H_S &= \frac{1}{2}\omega\sigma_z, \quad \omega = (\Omega^2 + \delta^2)^{1/2}. \end{aligned} \quad (6)$$

Taking into account Eqs. (4) to (6) we have for the Hamiltonian of a single NV spin interacting with the cantilever

$$\begin{aligned} \hat{H} &= \frac{1}{2}\omega\sigma_z + \omega_r(a^+a + 1/2) + \frac{\lambda}{2}(a^+ + a) \\ &\times [\cos\theta\sigma_z + \sin\theta(\sigma^+ + \sigma^-)]. \end{aligned} \quad (7)$$

Now we generalize the model given by Eq. (7) for the system of two spins (see Fig. 1). In the absence of an applied magnetic field the system is symmetric and the distance between the spins and the cantilever is equal  $h_1 = h_2 = h$ . However, due to the magnetic-field-induced cantilever deflection, the external magnetic field leads to an asymmetry  $h_1 \neq h_2$ . The imposed asymmetry  $\Delta h(B_0) = 2|h_1 - h_2|/(h_1 + h_2)$ ,  $0 < \Delta h(B_0) < 1$  can be quantified in terms of the amplitude of applied magnetic field  $B_0$  and the geometric and material characteristics of the cantilever, as outlined in Sec. III.

The amplitude of zero-point oscillations of the cantilever is of the order of  $a_0 = \sqrt{\hbar/(2m\omega_r)} \approx 5 \times 10^{-13}$  m, where  $m \approx 6 \times 10^{-17}$  kg is the resonator mass and  $\omega_r/(2\pi) \approx 4$  MHz is the first resonance frequency of the cantilever. Details on the cantilever are given in Sec. III.

This oscillation amplitude is definitely smaller than the asymmetry imposed by the external magnetic field, i.e.,  $\Delta h(B_0) \gg a_0$ . Nevertheless, the oscillations of the cantilever produce a varying magnetic field due to the attached magnetic tips, which is proportional to the oscillation amplitude. The key consequence of the asymmetry is that constants of the interaction between the spins and the magnetic tips  $\lambda_{1,2}$  are different for different spins and the Rabi transition frequencies  $\Omega_{1,2}$  are different as well. Therefore, the Hamiltonian of the system of two NV spins interacting with the deformed cantilever reads

$$\begin{aligned} \hat{H} &= \hat{H}_0 + \hat{V}, \\ \hat{H}_0 &= \frac{1}{2}\hbar\omega_1\sigma_1^z + \frac{1}{2}\hbar\omega_2\sigma_2^z + \hbar\omega_r(a^+a + 1/2), \\ \hat{V} &= \lambda_1(a^+ + a)\frac{1}{2}\hbar[\cos\theta_1\sigma_1^z + \sin\theta_1(\sigma_1^+ + \sigma_1^-)] \\ &\quad + \lambda_2(a^+ + a)\frac{1}{2}\hbar[\cos\theta_2\sigma_2^z + \sin\theta_2(\sigma_2^+ + \sigma_2^-)]. \end{aligned} \quad (8)$$

Here the term  $\hbar\omega_r(a^+a + 1/2)$  describes quantized oscillations of the cantilever,  $\tan\theta_{1,2} = -\frac{\Omega_{1,2}}{\delta}$  and  $\omega_{1,2} = \sqrt{\Omega_{1,2}^2 + \delta^2}$ , thereby denotes  $\Omega_{1,2}$  the Rabi frequency of the transition between the up and down spin states and  $\delta$  is the detuning between the microwave frequency and the intrinsic frequency of the spins. For realistic parameter values one finds  $\delta/\Omega < 1$ . Therefore, in the first approximation we assume the influence of the cantilever's asymmetry on the detuning to be minute as compared to the influence of the asymmetry on the Rabi frequency. In the symmetric case  $h_1 = h_2 = h$ ,  $\omega_1 = \omega_2 = \omega$  and  $\lambda_1 = \lambda_2 = \lambda$  the Hamiltonian (1) recovers the previously studied model.<sup>25,26</sup> The coupling constants between the spins and the cantilever has the form  $\lambda_{1,2} = g_S\mu_B G_{1,2}^m a_0$ , where  $g_S \approx 2$ ,  $\mu_B$  is the Bohr magneton and  $G_{1,2}^m = \frac{1}{2}|\vec{B}_{\text{tip}}|$  is the magnetic field gradient produced by the magnetic tips. Note that if the

asymmetry of the deformation of the cantilever is strong  $\Delta h(B_0) \sim 1$ , the distance between the magnetic tip and the nearest to the cantilever adjacent NV spin becomes very small  $z = h_1$  leading to the very large magnetic field gradient  $G_1^m = \frac{1}{h_1} |\vec{B}_{\text{tip}}|$  and to the large interaction constant  $\lambda_1 \approx g_S \mu_B G_{1,2}^m a_0 / h_1$ , while the coupling to the second spin becomes weak  $\lambda_2 \approx g_S \mu_B G_{1,2}^m a_0 / h_2$  and therefore the relation  $\frac{\lambda_1}{\lambda_2} = \frac{h_2}{h_1} \gg 1$  holds. This means that one can easily control the interaction between the magnetic tips and the NV spins simply by tuning the amplitude of the external magnetic field and thus controlling the deflection of the cantilever  $\Delta h(B_0)$ . In the symmetric case  $z = h_{1,2} = h$ ,  $\Delta h = 0$ ,  $G_1^m = G_2^m = G_m \approx 10^6$  [T/m] and realistic values of the parameters are  $h \approx 25$  nm,  $\lambda/(2\pi) \approx 0.1$  [MHz],  $\omega_r/(2\pi) \approx 5$  MHz. In what follows the interaction constant  $\lambda$  and the detuning between the cantilever frequency and the spin splitting  $\Delta = \omega_r - \omega$ ,  $\Delta \approx 2\lambda$  defines the time scale of the problem. A deviation of the values of the constants from the values corresponding to the symmetric case reads

$$\begin{aligned} \lambda_{1,2} &= \lambda(h_{1,2}) = \lambda \pm \Delta\lambda, & \Delta_{1,2} &= \Delta(h_{1,2}) = \Delta \pm \Delta\omega, \\ \omega_{1,2} &= \sqrt{(\Omega \pm \Delta\Omega)^2 + \delta^2} = \omega \pm \Delta\omega, \\ \Delta\omega &= \frac{\Omega^2}{\omega} (\Delta h), & \Delta\lambda &= \lambda \Delta h, & \Delta\Omega &= \Omega \Delta h. \end{aligned} \quad (9)$$

Estimates of the asymmetry parameter  $\Delta h(B_0)$  describing a deformation of the cantilever, for realistic materials and magnetic fields, follow in the next section.

### III. MAGNETIC-FIELD-INDUCED CANTILEVER DEFLECTION

We propose to deposit a magnetic film on the cantilever to explore alternatively magnetoelastic stress or magnetic-field-induced torque to induce a controlled cantilever deflection due to an external magnetic field  $B_0$ . In the first case we exploit the tendency of a film to develop magnetoelastic stress upon magnetization, and this stress induces a curvature of the thin cantilever substrate.<sup>35–37</sup> In the second case we exploit the magnetic torque<sup>38</sup>  $\vec{T} = \vec{m} \times \vec{B}_0$ , where  $\vec{m}$  is the total magnetic moment of the film, which is prepared to be oriented along the cantilever length ( $x$  direction), and  $\vec{B}_0$  is the applied magnetic field, oriented perpendicularly to the cantilever long axis ( $z$  direction). In this geometry a cantilever deflection along the  $z$  direction results.

For a quantitative determination of the resulting cantilever deflection we specify the cantilever dimensions as follows: length  $L = 3000$  nm, width  $w = 300$  nm, thickness  $t_s = 30$  nm. These parameters are well suited for the nanofabrication of Si cantilevers.<sup>39</sup> They also represent a valid scenario of stress-induced free two-dimensional bending due to the large length-to-width ratio.<sup>40</sup>

We assume that the cantilever is fabricated out of Si(100). We use the corresponding<sup>41</sup> Young modulus  $Y = 130$  GPa and Poisson ratio  $\nu = 0.279$ , and density  $\rho = 2.33 \times 10^3$  kg/m<sup>3</sup>. Such a cantilever has a mass of  $m = 6.29 \times 10^{-17}$  kg. Its first three resonance frequencies<sup>42</sup> are calculated from  $f_{\text{res}} = t_s \beta^2 [Y/(3\rho)]^{0.5} / (4\pi L^2)$  with  $\beta = (1.8751, 4.6941, 7.8548)$  as 4.02, 25.2, and 70.6 MHz. This cantilever has a negligible deflection at its end due to its own weight of  $3\rho L^4 / (2Y t_s^2) =$

$2.4 \times 10^{-15}$  m, which is more than six orders of magnitude smaller than the field-induced deflection, as described next.

#### I. Magnetoelastic-stress-induced cantilever deflection

Magnetoelastic stress is responsible for the change of length of a bulk sample upon magnetization, and the resulting strain is known as magnetostriction.<sup>35,43</sup> To illustrate the effect, we quote the magnetostriction data of bulk Ni:  $\lambda_{100} = -64.5 \times 10^{-6}$  and  $\lambda_{111} = -28.3 \times 10^{-6}$ . This means that a change of magnetization along the [100] ([111]) directions induces a lattice strain of the quoted magnitude in the Ni bulk sample along these directions, respectively.<sup>35,43</sup>

In films, a change of length of the film upon magnetization is not possible due to the bonding to the substrate, and the film develops a magnetoelastic stress. This stress induces a curvature of the substrate, which we exploit to deflect the end of the cantilever. The role of the external magnetic field is to induce a reorientation of the magnetization of the film from an in-plane  $M_x$  (external magnetic field off) to an out-of-plane direction  $M_z$  (external magnetic field on along the  $z$  direction). Such a reorientation of the magnetization direction of the film with thickness  $t_f$  induces a corresponding change of magnetoelastic stress. For a magnetization reversal along the axes of a cubic system the magnetoelastic coupling coefficient  $B_1$  enters.<sup>35</sup> It induces a change of the deflection of the free cantilever end of<sup>35</sup>

$$\text{defl}_{\text{me}} \Big|_{M_z}^{M_x} = \frac{3L^2 t_f (1 + \nu) B_1}{Y t_s^2}. \quad (10)$$

For definiteness, we assume a Ni film of thickness 10 nm and we take  $B_1$  of bulk Ni, 9.38 MJ/m<sup>3</sup>. A Ni film with bulk properties deposited on the top surface of the cantilever has a tendency to contract along the magnetization direction. Thus, the cantilever would be curved upwards for zero external field (magnetization in-plane along  $x$ ), and it would curve downward for magnetic field on (magnetization along  $z$ ). This is irrespective of the sign (along  $+z$  or along  $-z$ ) of the external field. The corresponding change of deflection results as  $\text{defl}_{\text{me}} = 27.7$  nm.

We note that the effective magnetoelastic coupling in thin films may deviate from its bulk value.<sup>35,36,44</sup> The physical origin of this deviation is a topic of current experimental and theoretical research. Our present understanding indicates that film strain, film thickness, and the film interfaces determine the resulting magnetoelastic coupling.<sup>44</sup> To illustrate the magnitude of the deviation from bulk values we refer to our measurements of an epitaxial film of 30 nm Ni on Cu(100) with a misfit of 1% which shows  $B_1 = 6.5$  MJ/m<sup>3</sup>, roughly 70% of its bulk value.<sup>44</sup>

The magnetoelastic stress shows a second-order dependence on the direction cosines  $\alpha_i$  of the magnetization direction.<sup>35,43</sup> If the magnetization of the film along direction  $z$  is driven by a sinusoidal magnetic field of magnitude  $\pm B_z$  of frequency  $f$ , the change of deflection occurs at frequency  $2f$ .

The required deflection can be adjusted by varying the experimental parameters  $t_f, t_s, L$ , accordingly. Note that the magnitude of the external magnetic field does not enter. The only requirement is that it is large enough to induce a magnetization reorientation from a single magnetic domain with magnetization along  $x$  to a single magnetic domain with



magnetization along  $z$ . If only a fraction of the film undergoes this magnetization reorientation, then the resulting deflection is correspondingly smaller.

The effective magnetic anisotropy of the magnetic film for in-plane as compared to an out-of-plane magnetization should be fairly small to achieve a substantial reorientation of magnetization with a limited magnitude of the external field. The maximum field is given by the requirement  $\mu_B B_0 < \hbar\omega_0$ , as pointed out above. This gives  $B_0 < 32.7$  mT. The magnetic anisotropy can be tuned by adjusting film thickness, morphology, interface modifications, multilayer structures, and film composition to achieve this goal. For example, a film close to an in-plane to out-of-plane spin reorientation transition can be prepared, and in this case a small out-of-plane field of 20 mT has been reported to saturate the sample magnetization along the  $z$  direction of a nine monolayer thin Ni film, where the magnetization reverts back to in-plane at zero field.<sup>45</sup>

## 2. Magnetic-torque-induced cantilever deflection

The deposition of a magnetic film on the cantilever opens also the possibility to exploit the torque induced by the external magnetic field along the  $z$  direction on the magnetic moments oriented along the  $x$  direction of the film to induce a cantilever deflection.<sup>46</sup> The magnetic torque  $\vec{T} = \vec{m} \times \vec{B}_0$  is proportional to both the total magnetic moment of the film  $m$  and the external magnetic field  $B_0$ . In this approach, the requirement for the magnetic anisotropy of the film is different as compared to the magnetoelastic-stress-induced deflection discussed above. Here, we require that the magnetic anisotropy of the film is large enough to ensure an in-plane magnetic moment in the presence of the out-of-plane field. This can be achieved by tuning the magnetic anisotropy differently as compared to the scenario discussed above. For example, by choosing a different buffer layer for the growth of the Ni film on the cantilever, lattice strain, and interface contributions to the magnetic anisotropy can be modified accordingly to achieve this goal.<sup>35,36</sup>

For definiteness we assume a magnetic moment of  $2\mu_B$  per film atom, and we refer to the atomic volume of bulk Ni,  $\rho_{\text{atomic}} = 1.096 \times 10^{-29} \text{ m}^{-3}$  to calculate the total number of Ni atoms in the film. We obtain for the torque-induced deflection<sup>38</sup>  $\text{defl}_{\text{torque}} = 4TL^2/(Ywt^3)$ , and from this we find

$$\text{defl}_{\text{torque}} = \frac{4t_f 2\mu_B B_0 L^3}{\rho_{\text{atomic}} Y t_s^3}. \quad (11)$$

We obtain a deflection of 5.2 nm for the parameters quoted above at a film thickness of  $t_f = 10$  nm for an external field  $B_0 = 10$  mT. Note that here the deflection is proportional to the external magnetic field, which allows a continuous control of the deflection. The magnetic film could be deposited on both sides of the cantilever, which would double the deflection in a given external magnetic field. Larger deflections at a given field are obtained by increasing the film thickness.

We foresee that the exploitation of the magnetoelastic stress for achieving a well-defined cantilever deflection is experimentally more challenging as compared to the exploitation of torque. One reason is the required exact tuning of the magnetic anisotropy. The torque approach is much more robust in that aspect, as only a sufficiently large energy barrier against out-of-

plane magnetization is needed. Already the shape anisotropy of out-of-plane magnetization fulfills this requirement.<sup>38</sup> Therefore we focus now on the torque-induced deflection.

To quantify the indirect interaction between the NV spins mediated by the interaction with the magnetic tip we consider again the asymmetry parameter  $\Delta h(B_0) = 2|h_1 - h_2|/(h_1 + h_2)$ . We calculate the magnetic tip–NV spin distances  $h_1, h_2$  as  $h_1 = h_0 - \text{defl}$  and  $h_2 = h_0 + \text{defl}$ , where  $h_0$  describes the symmetric case, i.e., the cantilever end is at the center position between both NV spins, which is realized for zero magnetic field for the torque-induced deflection. This gives the asymmetry parameter

$$\Delta h(B_0) = 2 \frac{\text{defl}_{\text{torque}}}{h_0} = \frac{16t_f \mu_B B_0 L^3}{h_0 \rho_{\text{atomic}} Y t_s^3}. \quad (12)$$

## IV. EFFECTIVE HAMILTONIAN: INDIRECT INTERACTION BETWEEN SPINS

The interaction of the NV spins with the magnetic tips leads to an indirect interaction between the NV spins. The Hamiltonian of the indirect interaction between the NV spins can be evaluated using the Fröhlich method<sup>47</sup>

$$\begin{aligned} \hat{H}_{\text{eff}} &= \frac{i}{2} \lambda^2 \int_{-\infty}^0 dt' [V(t'), V(0)], \\ \hat{V}(t) &= e^{-i\hat{H}_0 t} \hat{V} e^{i\hat{H}_0 t}. \end{aligned} \quad (13)$$

Taking into account Eqs. (8) and (13) in the rotating wave approximation we deduce

$$\begin{aligned} \hat{H}_{\text{eff}} &= -\frac{1}{4} \left\{ \sin^2 \theta_1 \frac{\lambda_1^2}{\Delta_1} (2n+1) \sigma_1^z + \sin^2 \theta_2 \frac{\lambda_2^2}{\Delta_2} (2n+1) \sigma_2^z \right\} \\ &\quad - \frac{1}{4} \sin \theta_1 \sin \theta_2 \left( \frac{\lambda_1 \lambda_2}{\Delta_1} + \frac{\lambda_1 \lambda_2}{\Delta_2} \right) (\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+), \\ n &= \langle \langle a^\dagger a \rangle \rangle, \quad \Delta_1 = \omega_r - \omega_1, \quad \Delta_2 = \omega_r - \omega_2. \end{aligned} \quad (14)$$

Taking into account that  $\sin \theta_{1,2} \approx \frac{\Omega}{\omega} (1 \pm \frac{\delta^2}{\omega^2} \Delta h)$ ,  $\frac{\delta^2}{\omega^2} \ll 1$ , and therefore,  $\sin \theta_{1,2} \approx \frac{\Omega}{\omega} = \sin \theta$  we can rewrite the interaction constant in terms of the asymmetry parameter

$$\begin{aligned} &\lambda_1 \lambda_2 \sin^2 \theta \frac{\Delta_1 + \Delta_2}{\Delta_1 \Delta_2} \\ &\approx \frac{2\lambda^2 \Omega^2}{\omega^2 \Delta} \cdot \frac{(1 - (\Delta h)^2)}{1 - \left(\frac{\Omega^2}{\omega \Delta}\right)^2 (\Delta h)^2} \\ &= \frac{2\lambda^2}{\omega} \left(\frac{\Omega^2}{\omega \Delta}\right) \left\{ 1 + \left[ \left(\frac{\Omega^2}{\omega \Delta}\right)^2 - 1 \right] (\Delta h)^2 \right\}. \end{aligned} \quad (15)$$

We note that Eq. (15) is derived using perturbation theory and the validity of the perturbation theory requires  $\lambda/\Delta < 1$ . From Eq. (15) we see that the asymmetry can enhance the interaction between the spins if

$$\alpha = \frac{\Omega^2}{\omega \Delta} > 1. \quad (16)$$

In the opposite case,

$$\alpha < 1, \quad (17)$$

the asymmetry lowers the interaction strength. Since the variance of the parameters is relatively large<sup>25,26</sup>  $\frac{\omega_r}{2\pi} \approx 1\text{--}5$  MHz,  $\Omega \approx 0.1\text{--}10$  MHz,  $\delta \approx 0.01\text{--}0.1$  MHz, both cases given by Eqs. (16) and (17) can be realized.

In particular from Eq. (15) we see that the interaction strength between spins depends on the asymmetry  $\Delta h$  and the dimensionless parameter  $\alpha = \frac{\Omega^2}{\omega\Delta}$ . Considering standard values of the parameters<sup>25,26</sup>  $\omega \approx \Omega, \lambda = \frac{\Delta}{2}$  from Eq. (15) for the interaction strength between spins we get:  $\lambda[1 + (\alpha^2 - 1)(\Delta h)^2]$ . We see that the dependence of the interaction strength between the NV spins on the asymmetry parameter  $\Delta h$  is not trivial. For small values of the parameter  $\alpha < 1$  the asymmetry arising from the deformation of the cantilever leads to the reduction of interaction strength, while for  $\alpha > 1$  the asymmetry increases the interaction strength. In particular, for the values  $\alpha = 3, \Delta h = 0.5$  the interaction between NV spins is three times larger than the interaction strength in the symmetric case  $\sim 3\lambda$ . A stronger interaction between spins means larger entanglement. Therefore controlling the spin coupling strength we can influence the degree of entanglement as well. From the physical point of view small values of the parameter  $\alpha \approx \frac{\Omega}{\omega_r - \Omega} < 1$  correspond to a large detuning between the oscillation frequency of the cantilever  $\omega_r$  and the Rabi frequency  $\Omega$ s, while for  $\alpha > 1$  we have the opposite case.

### V. INFLUENCE OF ASYMMETRY ON THE ENTANGLEMENT DEGREE

To study the influence of the asymmetry on the degree of entanglement, we directly solve the Schrödinger equation corresponding to the Hamiltonian (14)

$$i \frac{d|\psi\rangle}{dt} = \hat{H}_{\text{eff}}|\psi\rangle, \\ |\psi\rangle = C_1(t)|\Phi^+\rangle + C_2(t)|\Phi^-\rangle + C_3(t)|\psi^+\rangle + C_4(t)|\psi^-\rangle. \quad (18)$$

Here  $|\Phi^\pm\rangle$  and  $|\psi^\pm\rangle$  are Bell states.<sup>48–50</sup> Taking into account Eqs. (14) and (18) for the resolution coefficients we obtain

$$C_1(t) = C_1(0) \cos(At) - C_2(0) \sin(At), \\ C_2(t) = C_2(0) \cos(At) - C_1(0) \sin(At), \\ C_3(t) = \frac{D^2 C_3(0) - D(F+B)C_4(0)}{2(F+B)F} e^{iFt} \\ + \frac{D^2 C_3(0) + D(F-B)C_4(0)}{2(F-B)F} e^{-iFt}, \\ C_4(t) = \frac{-DC_3(0) + (F+B)C_4(0)}{2F} e^{iFt} \\ + \frac{DC_3(0) + (F-B)C_4(0)}{2F} e^{-iFt}, \quad (19)$$

with the following notations

$$A = \frac{\lambda^2}{2\omega} (2n+1) \frac{\Omega^2}{\omega\Delta} \left[ 1 - \left( \frac{\Omega^2}{\omega\Delta} - 1 \right) (\Delta h)^2 \right], \\ B = -\frac{\lambda^2 \Omega^2}{2\omega^2 \Delta} \left[ 1 - \left( \frac{\Omega^2}{\omega\Delta} - 1 \right) (\Delta h)^2 \right], \\ D = -\frac{\lambda^2}{2\omega} (2n+1) \frac{\Omega^2}{\omega\Delta} \left( \frac{\Omega^2}{\omega\Delta} - 1 \right) \Delta h, \\ F = \sqrt{B^2 + D^2}. \quad (20)$$

Taking into account Eqs. (18) and (19) we can quantify the entanglement via the following expression<sup>48</sup>

$$C[|\psi(t)\rangle] = |\langle \psi^*(t) | \sigma_y \otimes \sigma_y | \psi(t) \rangle|. \quad (21)$$

As a result, after straightforward but laborious calculations for concurrence we deduce

$$|C(t)| = \left| \left[ C_1^2(0) - C_2^2(0) \right] \cos 2At + e^{-2iFt} \left( \frac{[-DC_3(0) + (F+B)C_4(0)]^2}{4F^2} \right) - e^{-2iFt} \left( \frac{[D^2 C_3(0) - D(F+B)C_4(0)]^2}{4F^2(F+B)^2} \right) \right. \\ \left. + e^{2iFt} \left( \frac{[DC_3(0) + (F-B)C_4(0)]^2}{4F^2} \right) - e^{2iFt} \left( \frac{[D^2 C_3(0) + D(F-B)C_4(0)]^2}{4F^2(F-B)^2} \right) \right. \\ \left. + \frac{[-DC_3(0) + (F+B)C_4(0)][DC_3(0) + (F-B)C_4(0)]}{2F^2} \right. \\ \left. - \frac{[DC_3(0) - D(F+B)C_4(0)][D^2 C_3(0) + D(F-B)C_4(0)]}{2F^2(F-B)^2} \right|. \quad (22)$$

### VI. RESULTS

Following Eq. (22) the concurrence is plotted in Figs. 3, 4, and 5 for a variation of two parameters  $\alpha$  and  $\Delta h$ .

The contour plot of  $|C(t)|$  is presented in Fig. 6.

From Fig. 4 we see that for small values of the parameter  $\alpha = 0.25$ , the increase of the asymmetry  $\Delta h$  leads to a smaller concurrence, while as we see from Figs. 3, 5, and 6, for large  $\alpha > 1$  concurrence is increased with asymmetry. In particular, this effect is more evident on the contour plot of Fig. 6 which defines the domains of maximal and minimal

concurrence as a function of parameters  $\Delta h, \alpha$ . We see that for  $\alpha < 0.5$  concurrence is maximal for small asymmetry  $\Delta h < 0.25$ , while for  $\alpha > 1.5$  concurrence is maximal for large asymmetry  $\Delta h > 0.3$ . The explanation of the observed effect is that for  $\alpha < 1$  the increase of the asymmetry  $\Delta h$  reduces the coupling strength between spins, see Eq. (15), while for  $\alpha > 1$  the increase of the asymmetry  $\Delta h$  increases the coupling strength between spins. This means that concurrence is larger for a larger interaction constant between the NV spins.

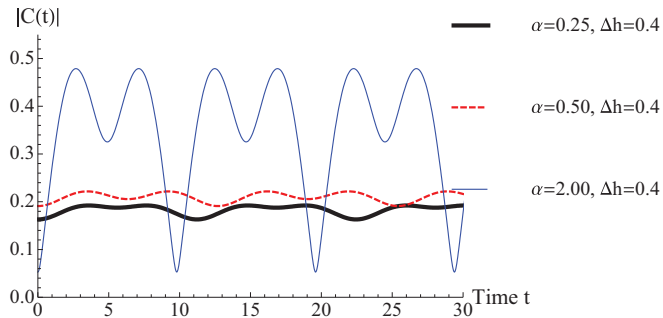


FIG. 3. (Color online) Time evolution of the concurrence  $|C(t)|$  for the given values of  $\alpha$  and  $\Delta h$ . Other parameters read  $n = 1$ ,  $C_1 = C_2 = C_3 = C_4 = 1/2$ . Time scale corresponds to the microsecond.

## VII. CONCLUSION

One of the main challenges for the NV spin-based nanoelectromechanical resonator has been the achievement of a high controlled degree of entanglement and a strong coupling between NV spins. With this in mind, we proposed in this work a new type of nanoelectromechanical resonator, the functionality of which is based on the NV spin controlled by an external magnetic field. In particular, we suggest to deposit a thin magnetic Ni film on the Si(100) cantilever to exploit alternatively magnetoelastic stress or magnetic-field-induced torque for inducing a controlled cantilever deflection upon acting with an external magnetic field. We have shown that, depending on the values of parameter  $\alpha = \frac{\Omega^4}{\omega^2 \Delta^2}$ , the induced asymmetry of the cantilever deflection substantially modifies the characteristics of the system. In particular we demonstrated that if  $\alpha > 1$  the asymmetry enhances the strength of the interaction between the NV spins is at least three times  $\sim 3\lambda$ , where  $\lambda = 100$  kHz is the maximal coupling strength between NV spins for the symmetric model reported by the authors of Ref. 25. However, for  $\alpha < 1$  the asymmetry reduces the interaction strength. In addition we found that for  $\alpha > 1$  entanglement is maximum for the case of a large asymmetry  $\Delta h$ , while for  $\alpha < 1$  the entanglement is maximal in the small asymmetry case (see Fig. 6). The values of the parameter  $\alpha$  can be changed efficiently via the change of the detuning between the oscillation frequency of the cantilever and the

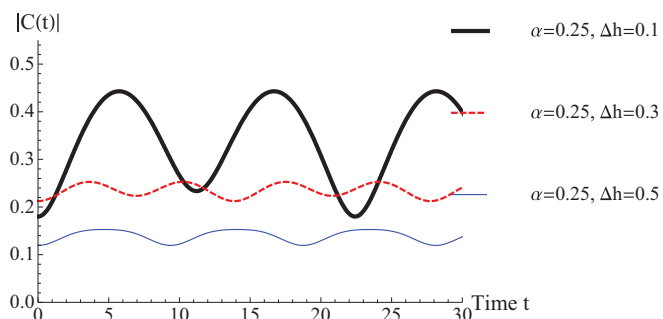


FIG. 4. (Color online) Time evolution of the concurrence  $|C(t)|$  for the given values of  $\alpha$  and  $\Delta h$ . Other parameters read  $n = 1$ ,  $C_1 = C_2 = C_3 = C_4 = 1/2$ . Time scale corresponds to the microsecond.

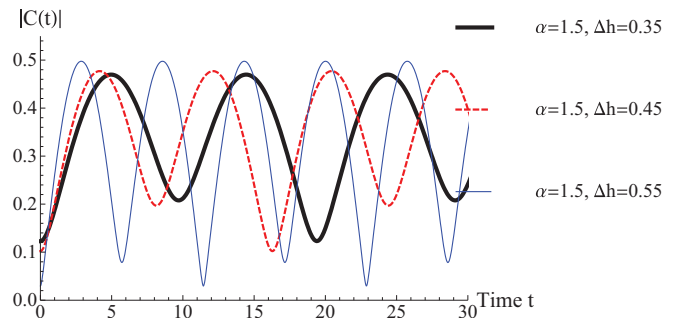


FIG. 5. (Color online) Time evolution of the concurrence  $|C(t)|$  for the given values of  $\alpha$  and  $\Delta h$ . Other parameters read  $n = 1$ ,  $C_1 = C_2 = C_3 = C_4 = 1/2$ . Time scale corresponds to the microsecond.

spin splitting frequency  $\Delta = \omega_r - \omega$ . This can be used as an effective tool for a practical implementation of our theoretical proposal for controlling the entanglement and the interaction strength between NV spins in the experiment.

## ACKNOWLEDGMENTS

The financial support by the Deutsche Forschungsgemeinschaft (DFG) through SFB 762, Contract No. BE 2161/5-1, 15 Grant No. KO-2235/3 is gratefully acknowledged. This work is also supported by the National Science Center in Poland as a research project in the years 2011–2014.

## APPENDIX

The interaction of the NV spins with the magnetic tips leads to an indirect interaction between the NV spins. The Hamiltonian of the indirect interaction between the NV spins can be evaluated using the Fröhlich method.<sup>47</sup> This method is widely used in studies on the magnetic resonance for the evaluation of the nondirect interaction between two different subitems mediated via a third subsystem, e.g., the interaction between the electron and the nuclear spins with phonons leads

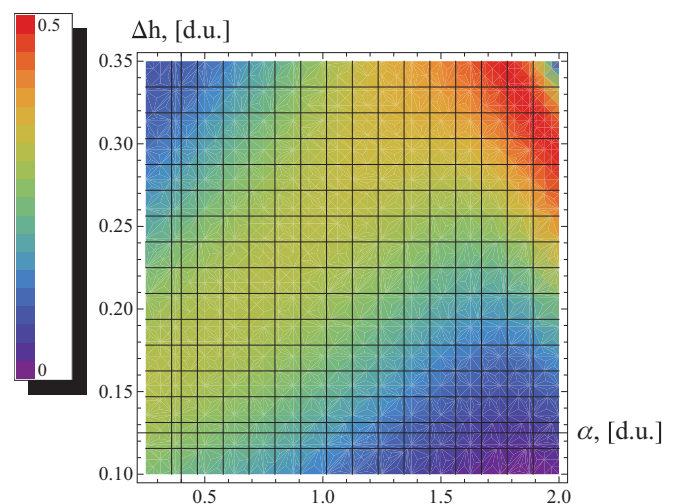


FIG. 6. (Color online) Contour plot of the concurrence  $|C(t)|$  for  $t = 13.3$ . Other parameters are  $n = 1$ ,  $C_1 = C_2 = C_3 = C_4 = 1/2$ .

to the nondirect interaction between electron and nuclear spins. In our case the NV spin interacts with a magnetic tip and the magnetic tip interacts with a second NV spin. This leads to an indirect interaction between the NV spins. The idea of the Fröhlich method is the following: Consider a system with the Hamiltonian  $\hat{H}_0$  that consists of two noninteracting subsystems ( $S_1$  and  $S_2$ ) coupled to a further system. Let  $\lambda\hat{V}$  be the coupling of both subsystems to this third system. The task is now to find the indirect effective interaction between  $S_1$  and  $S_2$  due to  $\lambda\hat{V}$ . To this end one performs a (unitary) transformation  $e^{-i\hat{S}}\hat{H}e^{i\hat{S}}$  where the operator  $\hat{S}$  is found from the relation  $\lambda\hat{V} + [\hat{H}_0, \hat{S}] = 0$  and  $H$  is the total Hamiltonian.

The effective interacting Hamiltonian reads

$$\hat{H}_{\text{eff}} = \frac{i}{2}\lambda^2 \int_{-\infty}^0 dt' [V(t'), V(0)],$$

$$\hat{V}(t) = e^{-i\hat{H}_0 t} \hat{V} e^{i\hat{H}_0 t}. \quad (\text{A1})$$

In our case the free Hamiltonian  $\hat{H}_0$  and the interaction  $\hat{V}$  are defined by Eq. (8). To derive Eq. (14) we should insert Eq. (8) in Eq. (A1) and adopt the usual rotating wave approximation. This means that after the integration over the time we neglect terms proportional to  $1/(\omega_r + \omega_{1,2})$  since they are smaller compared to the terms proportional to  $1/(\omega_r - \omega_{1,2})$ .

- <sup>1</sup>A. K. Naik, M. S. Hanay, W. K. Hiebert, X. L. Feng, and M. L. Roukes, *Nature Nanotechnology* **4**, 445 (2009).
- <sup>2</sup>A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, M. Lenander, E. Lucero, M. Neeley, D. Sank, H. Wang, M. Weides, J. Wenner, John M. Martinis, and A. N. Cleland, *Nature (London)* **464**, 697 (2010).
- <sup>3</sup>A. H. Safavi-Naeini, T. P. M. Alegre, J. Chan, M. Eichenfield, M. Winger, Q. Lin, J. T. Hill, D. Chang, and O. Painter, *Nature (London)* **472**, 69 (2011).
- <sup>4</sup>K. Stannigel, P. Rabl, A. S. Sorensen, P. Zoller, and M. D. Lukin, *Phys. Rev. Lett.* **105**, 220501 (2010).
- <sup>5</sup>A. H. Safavi-Naeini and O. Painter, *New J. Phys.* **13**, 013017 (2011).
- <sup>6</sup>S. Camerer, M. Korppi, A. Jöckel, D. Hunger, T. W. Hänsch, and P. Treutlein, *Phys. Rev. Lett.* **107**, 223001 (2011).
- <sup>7</sup>M. Eichenfield, J. Chan, R. M. Camacho, K. J. Vahala, and O. Painter, *Nature (London)* **462**, 78 (2009).
- <sup>8</sup>A. H. Safavi-Naeini, J. Chan, J. T. Hill, Thiago P. Alegre, A. Krause, and O. Painter, *Phys. Rev. Lett.* **108**, 033602 (2012).
- <sup>9</sup>N. Brahm, T. Botter, S. Schreppler, D. W. C. Brooks, and D. M. Stamper-Kurn, *Phys. Rev. Lett.* **108**, 133601 (2012).
- <sup>10</sup>A. Nunnenkamp, K. Borkje, and S. M. Girvin, *Phys. Rev. A* **85**, 051803 (2012).
- <sup>11</sup>F. Ya. Khalili, H. Miao, H. Yang, A. H. Safavi-Naeini, O. Painter, and Y. Chen, *Phys. Rev. A* **86**, 033840 (2012).
- <sup>12</sup>C. P. Meaney, R. H. McKenzie, and G. J. Milburn, *Phys. Rev. E* **83**, 056202 (2011).
- <sup>13</sup>J. Atalaya, A. Isacsson, and M. I. Dykman, *Phys. Rev. B* **83**, 045419 (2011).
- <sup>14</sup>P. Rabl, *Phys. Rev. B* **82**, 165320 (2010).
- <sup>15</sup>S. V. Prants, *J. Phys. A* **44**, 265101 (2011).
- <sup>16</sup>M. Ludwig, K. Hammerer, and F. Marquardt, *Phys. Rev. A* **82**, 012333 (2010).
- <sup>17</sup>T. L. Schmidt, K. Borkje, C. Bruder, and B. Trauzettel, *Phys. Rev. Lett.* **104**, 177205 (2010).
- <sup>18</sup>R. B. Karabalin, M. C. Cross, and M. L. Roukes, *Phys. Rev. B* **79**, 165309 (2009).
- <sup>19</sup>L. Chotorlishvili, A. Ugulava, G. Mchedlishvili, A. Komnik, S. Wimberger, and J. Berakdar, *J. Phys. B: At. Mol. Opt. Phys.* **44**, 215402 (2011).
- <sup>20</sup>S. N. Shevchenko, A. N. Omelyanchouk, and E. Il'ichev, *Low Temp. Phys.* **38**, 283 (2012).
- <sup>21</sup>Y.-X. Liu, A. Miranowicz, Y. B. Gao, J. Bajer, C. P. Sun, and F. Nori, *Phys. Rev. A* **82**, 032101 (2010).
- <sup>22</sup>S. N. Shevchenko, S. Ashhab, and F. Nori, *Phys. Rep.* **492**, 1 (2010).
- <sup>23</sup>D. Zueco, G. M. Reuther, S. Kohler, and P. Hänggi, *Phys. Rev. A* **80**, 033846 (2009).
- <sup>24</sup>G. Z. Cohen and M. Di Ventra, *Phys. Rev. B* **87**, 014513 (2013).
- <sup>25</sup>P. Rabl, P. Cappellaro, M. V. Gurudev Dutt, L. Jiang, J. R. Maze, and M. D. Lukin, *Phys. Rev. B* **79**, 041302(R) (2009).
- <sup>26</sup>L.-G. Zhou, L. F. Wei, M. Gao, and X.-B. Wang, *Phys. Rev. A* **81**, 042323 (2010).
- <sup>27</sup>O. Arcizet, V. Jacques, A. Siria, P. Poncharal, P. Vincent, and S. Seidelin, *Nat. Phys.* **7**, 879 (2011).
- <sup>28</sup>D. Rugar, R. Budakian, H. J. Mamin, and B. W. Chui, *Nature (London)* **430**, 15 (2004).
- <sup>29</sup>P. Treutlein, *Science* **335**, 1584 (2012).
- <sup>30</sup>S. Kolkowitz, A. C. Bleszynski-Jayich, Q. P. Unterreithmeier, S. D. Bennett, P. Rabl, J. G. E. Harris, and M. D. Lukin, *Science* **335**, 1603 (2012).
- <sup>31</sup>Z.-H. Wang and V. V. Dobrovitski, *Phys. Rev. B* **84**, 045303 (2011).
- <sup>32</sup>J. Wrachtrup and F. Jelezko, *J. Phys.: Condens. Matter* **18**, S807 (2006).
- <sup>33</sup>L. Chotorlishvili and Z. Toklikishvili, *Phys. Lett. A* **372**, 2806 (2008).
- <sup>34</sup>H. Yoo and J. Eberly, *Phys. Rep.* **118**, 240 (1985).
- <sup>35</sup>D. Sander, *Rep. Prog. Phys.* **62**, 809 (1999).
- <sup>36</sup>D. Sander, *J. Phys.: Condens. Matter* **16**, R603 (2004).
- <sup>37</sup>D. Sander, *Sensors* **8**, 4466 (2008).
- <sup>38</sup>Th. Höpfl, D. Sander, and J. Kirschner, *Rev. Sci. Instrum.* **72**, 1495 (2001).
- <sup>39</sup>D. Saya, K. Fukushima, H. Toshiyoshi, G. Hashiguchi, H. Fujita, and H. Kawakatsu, *Sensors and Actuators A* **95**, 281 (2002).
- <sup>40</sup>K. Dahmen, H. Ibach, and D. Sander, *J. Magn. Magn. Mater.* **231**, 74 (2001).
- <sup>41</sup>W. A. Brantley, *J. Appl. Phys.* **44**, 534 (1973).
- <sup>42</sup>D. Sarid, *Exploring Scanning Probe Microscopy with Mathematica* (Wiley, New York, 1997).
- <sup>43</sup>Ch. Kittel, *Rev. Mod. Phys.* **21**, 541 (1949).
- <sup>44</sup>D. Sander and J. Kirschner, *Phys. Status Solidi B* **248**, 2389 (2011).
- <sup>45</sup>R. Denk, M. Hohage, and P. Zeppenfeld, *Phys. Rev. B* **79**, 073407 (2009).
- <sup>46</sup>D. Sander, R. Skomski, C. Schmidhals, A. Enders, D. Reuter, and J. Kirschner, *J. Phys. D: Appl. Phys.* **31**, 663 (1998).
- <sup>47</sup>H. Fröhlich, *Proc. R. Soc. London, Ser. A* **215**, 291 (1952); P. Tarasewicz and D. Baran, *Phys. Rev. B* **73**, 094524 (2006); C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1987).
- <sup>48</sup>F. Mintert, A. Carvalho, M. Kus, and A. Buchleitner, *Phys. Rep.* **415**, 207 (2005).
- <sup>49</sup>L. Amico, R. Fazio, A. Osterloh, and V. Vedral, *Rev. Mod. Phys.* **80**, 517 (2008).
- <sup>50</sup>W. K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).