# APPARATUS AND DEMONSTRATION NOTES

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# Demonstration of different bending profiles of a cantilever caused by a torque or a force

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A simple experiment is described to demonstrate the different deflected shapes assumed by a cantilever due to a torque or a force acting on its end. An optical deflection technique is used to show that different shapes appear in the cantilever even if the torque or force causes an identical displacement of the cantilever end.
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## I. INTRODUCTION

We describe a demonstration experiment which shows that a torque or a force acting on the free end of a beam clamped at the opposite end lead to qualitatively different bending curves. Although the different bending profiles can easily be derived from the classical theory of elasticity, it is very instructive to observe the difference directly. To this end a simple optical deflection technique is used to monitor the bending of a cantilever on which a permanent magnet is mounted. In the presence of a homogeneous magnetic field a pure torque is applied and, in the presence of a suitable nonhomogeneous magnetic field, a pure force is applied.

## II. MOTIVATION

To elucidate the difference between a torque and a force acting on a bending beam, the following experiment can be performed first. A plate of suitable dimensions, e.g., an aluminum sheet of $50\,\mathrm{cm}\times 5\,\mathrm{cm}\times 1\,\mathrm{mm}$, is rigidly clamped at one end using a vise on a table-top, as shown in Fig. 1. Near the free end of the plate, a threaded rod is fixed using nuts so that it stands out about 7 cm on both sides of the plate. With pieces of yarn fixed at the two ends of this rod, one can exert a torque at the end of the plate by pulling the pieces of yarn in opposite directions parallel to the plate. A force on the end of the bending beam can be exerted by pulling the yarn downwards. Visual inspection of the bent aluminum sheet reveals that a force bends it into a shape where most of the curvature occurs close to the clamped end; near the free end, an almost constant slope is observed. In the case of a torque, the curvature is distributed along the whole length of the aluminum sheet, i.e., one observes a change of slope also near the free end where the torque is applied. This experiment illustrates the difference between the bent shapes due to torques and forces, but one gets only a qualitative feeling for the difference. For more quantitative observations, the following simple optical deflection experiment is proposed.

## III. EXPERIMENT

The basic idea of the experiment is that a permanent magnet can experience both a torque or a force in a magnetic field.

If a magnet with a magnetic moment $\vec{m}$ is placed in a homogeneous magnetic field of flux density $\vec{B}$, then a torque $\vec{T}=\vec{m}\times\vec{B}$ acts on it, as in the familiar case of a compass needle. If a magnet is exposed to an inhomogeneous magnetic field, then a force $\vec{F}=(\vec{m}\cdot\vec{\nabla})\vec{B}$ acts on it. This principle is exploited in force magnetometers that are used to measure the total magnetic moment of a sample.[1]

The magnetic fields required for our experiment are produced by a pair of coils in a Helmholtz-type setup. Each coil is wound on the aluminum rim of a bicycle wheel (nominal diameter 24 in. and measured diameter 56 cm) and consists of 70 turns of insulated copper wire of diameter 1.7 mm. The resistance of each coil is 0.96 $\Omega$. The coils are mounted parallel to each other a distance of 27 cm apart.

A homogeneous field is generated when a current runs with the same sense of winding through both coils. At 10 A this leads to a nearly homogeneous field of flux density 2.27 mT at the region midway between the coils. Along the axis of the coils, the magnetic flux density varies by less than 0.01 mT for a several cm displacement to either side of the center point.

An inhomogeneous field is produced by passing a current with the opposite sense of winding through the two coils. In

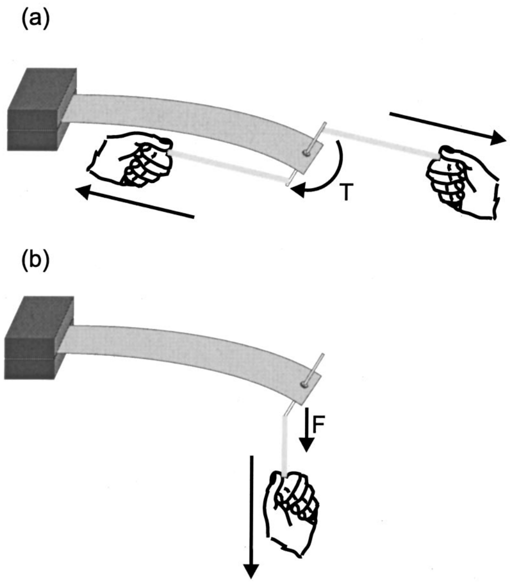

Fig. 1. Simple first experiment: A torque $T$ (a) and a force $F$ (b) are applied to the end of a bending beam, inducing different shapes in the beam.

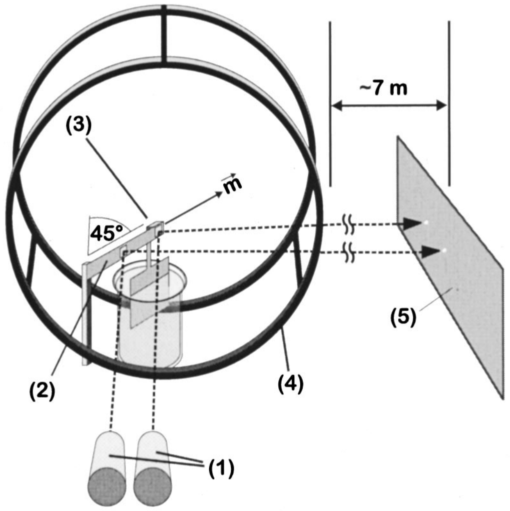

Fig. 3. Light rays from the two lasers (1) experience different deflections at the Si pieces on the beam (2) depending on whether a torque or a force is acting on permanent magnet (3). The torque and the force are produced by a pair of coils (4). The reflected light is observed on a white screen (5) about 7 m from the experiment.

that case, the homogeneous part of the field is compensated to nearly zero, but a field gradient occurs at the center of the pair. At a current of 10 A this field gradient amounts to 9 $\mathrm{mT\,m^{-1}}$.

If a cantilever with a permanent magnet attached near its free end is positioned midway between the coils, a torque or a force can be exerted on it by applying a homogeneous or an inhomogeneous magnetic field, respectively. To examine both torque and force effects without having to reorient the cantilever, it is useful to place the cantilever axis at an angle of 45° to the axis of the pair of coils. A permanent magnet is mounted at the free end of the cantilever with its magnetic axis along the cantilever axis. The apparatus is illustrated in Fig. 2. With this arrangement, a pure torque or pure force can be produced in the following ways: If a homogeneous field is applied, a pure torque acts on that component of $\vec{m}$ which is oriented perpendicular to the axis of the coils. In the case of an inhomogeneous field, a pure force acts on the component of $\vec{m}$ which is parallel to the axis of the coils.

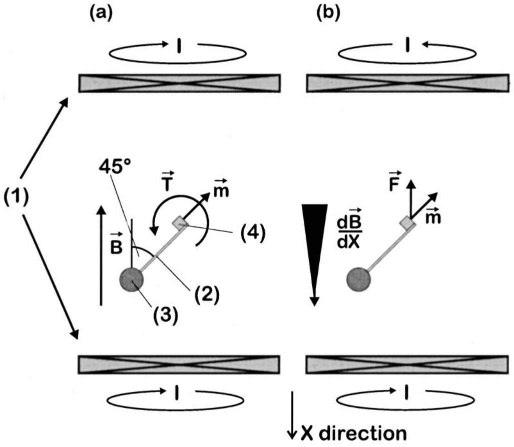

Fig. 2. Schematic of the experiment (top view). Two coils produce (a) an homogenous or (b) an inhomogeneous field. This leads to a torque $T$ or a force $F$ on a permanent magnet (4) fixed at the free end of a bending beam (2), which is rotated by 45° from the axis of the pair of coils. The magnetization of the magnet is in the direction of the beam. The principal components shown are (1) coils, (2) bending beam with two glued Si pieces, (3) clamping, and (4) permanent magnet. The direction of the field gradient $d\vec{B}/dX$ is indicated by the large arrow in (b).

A brass cantilever ($15\,\mathrm{cm}\times1.5\,\mathrm{cm}\times150\,\mu\mathrm{m}$) is used in the experiment, as shown in Fig. 3. A permanent magnet[2] (NdFeB, $8\,\mathrm{mm}\times8\,\mathrm{mm}\times6\,\mathrm{mm}$) is glued to one side of the brass beam with its direction of magnetization oriented along the beam. Two Si pieces ($7\,\mathrm{mm}\times7\,\mathrm{mm}$) are glued to the other side of the beam, one at the free end and one in the middle. These two Si pieces serve as mirrors, so that the deflection of the cantilever can be monitored by reflecting light from two 0.95 mW He–Ne lasers[3] positioned about 30 cm from the cantilever. To facilitate observation by a large audience, the reflected rays are viewed on a white screen about 7 m from the cantilever. For easy adjustment, the lasers are mounted on two gimbal mounts attached to an optical bench. To reduce and dampen vibrations of the cantilever, a paddle ($5\,\mathrm{cm}\times3\,\mathrm{cm}$) is fixed to it by a narrow connection to its rear side and this paddle is immersed in a cup of water. The paddle is made from a 250 $\mu$m sheet of brass and does not noticeably influence the resulting bending profile of the beam.

## IV. EXECUTION

In our experiment a current of 10 A produces enough torque or force (depending on the way the coils are connected together) to deflect the free end of the cantilever by a few mm.

The deflection $w_T(x)$ of a cantilever of length $l$ under the influence of a torque $T$ acting on its free end is given by the second order polynomial[4]

$$w_T(x)=\frac{Tl^2}{2EI_y}\left[1-\frac{2x}{l}+\left(\frac{x}{l}\right)^2\right], \tag{1}$$

where $E$ is Young's modulus of the beam material and $I_y$ is the second moment of area of the beam cross section about the neutral axis. (For rectangular beam cross sections, $I_y$

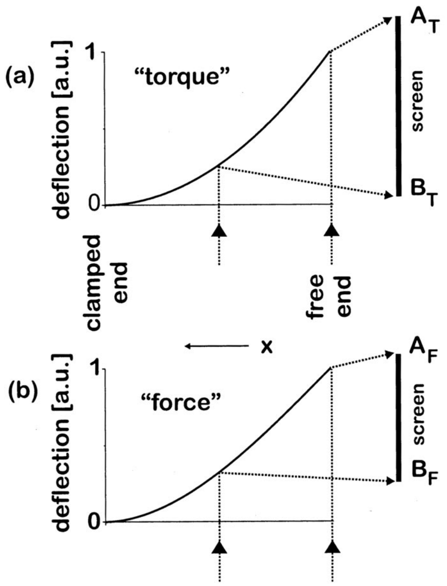

Fig. 4. The result of the demonstration experiment. Two laser beams (dotted lines) are reflected at the middle and at the free end of the cantilever. In the case of a pure torque acting on the free end of the cantilever (a), the laser beams reflected from the free end and the middle of the cantilever, are, respectively, more deflected and less reflected than in the case of a pure force (b), although in both cases the distance displacement of the cantilever at its free end is identical. It follows that the distance $A_TB_T$ is greater than the distance $A_FB_F$ .

$=ba^3/12$, where $b$ is the width, and $a$ is the thickness.) The distance $x$ is measured from the free end of the cantilever towards the clamp (*i.e., x=0 corresponds to the free end*).

The deflection $w_F(x)$ for a force $F$ acting on the free end is given by the third order polynomial[4]

$$w_F(x)=\frac{Fl^3}{6EI_y}\left[2-\frac{3x}{l}+\left(\frac{x}{l}\right)^3\right]. \tag{2}$$

The two shapes of the bent cantilever together with the incident and the reflected laser beams are shown in Fig. 4. If a cantilever of length 15 cm is deflected 5 mm at its free end, then the laser beam reflected at this end changes its position on the screen (7 m away) by 93 cm if a torque is acting and by 70 cm in the case of a force. The laser beam reflected at the middle undergoes a smaller deflection. In the case of a force acting on the free end, the reflected beam from the middle of the cantilever changes its position more than in the case of a torque. This leads to the result that the distance between the two reflected beams on the observation screen is larger for the application of a torque than for the application of a force. This result is sketched in Fig. 4.

## V. DISCUSSION

The displacements of the reflected laser spots are very large, and this experiment is well suited to large lecture rooms. To obtain the most impressive effect, it is advantageous to adjust the laser so that before bending the beam, the two reflected laser spots at the screen are at the same horizontal height and are separated by about 20 cm. To point out the displacement of the spots, the spot positions should be marked by colored paper arrows at their original and at their final positions.

Depending on the preference of the lecturer and the context, the experiment can be performed in two ways: one can adjust to obtain the same displacement of the end of the beam or one can adjust it to obtain the same displacement of one of the spots. Both possibilities show very clearly the qualitatively different bending profiles.

Beyond this tutorial demonstration, the bending of cantilevers is crucial for our understanding of cantilever magnetometers. These magnetometers are used in ultrahigh vacuum conditions to measure magnetic moments of ultrathin films deposited as a single atomic layer onto single crystal substrates.[5–8] In these experiments, the interaction of an external magnetic field with the magnetic moments of the film induces a mechanical torque that leads to tiny ($10^{-9}$ m) deflections of a cantilever substrate. Capacitive[5] or optical[8] detection of this deflection has been applied to measure magnetic moments with a sensitivity corresponding to the total magnetic moment of $10^{13}$Fe atoms (≈1 ng of Fe).

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[1] S. Foner, "Review of mangetometry," IEEE Trans. Magn. **17**, 3558–3363 (1981).

[2] IBS Magnet, Model No. NE88, Kurfürstenstr. 92, D-12105 Berlin, Germany.

[3] JDS Uniphase, Model No. 1108P, www.jdsunph.com.

[4] R. J. Roark, *Formulas for Stress and Strain*, 6th ed. (McGraw-Hill, Singapore, 1989).

[5] M. Weber, R. Koch, and K. H. Rieder, "UHV cantilever beam technique for quantitative measurements of magnetism," Phys. Rev. Lett. **73**, 1166–1169 (1994).

[6] D. Sander, A. Enders, and J. Kirschner, "Magnetization, magnetostriction and film stress of Fe monolayers on W(100)," IEEE Trans. Magn. **34**, 2015–2017 (1998).

[7] Th. Hopfl, Ph.D. thesis, Martin-Luther-Universität, Halle, Wittenberg, 2000.

[8] Th. Höpfl, D. Sander, H. Höche, and J. Kirschner, "Ultrahigh vacuum cantilever magnetometry with standard size single crystal substrates," Rev. Sci. Instrum. **72**, 1495–1501 (2001).