Journal of Magnetism and Magnetic Materials 321 (2009) 906–908

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Two-dimensional electron gas in a periodic magnetic field

C. Lacroix [a,*], M. Taillefumier [a,b,c], V.K. Dugaev [d,e], B. Canals [a], P. Bruno [b,f]

[a] Institut Néel, CNRS-UJF, BP 166, 38042 Grenoble Cedex 9, France
[b] Max-Planck-Institut für Mikrostruktur Physik, Weinberg 2, 06120 Halle, Germany
[c] Department of Physics, Norwegian University of Science and Technology, N-7491, Trondheim, Norway
[d] Department of Physics, Rzeszow University of Technology, Al. Powstancow Warszawy 6, 35-959 Rzeszow, Poland
[e] Department of Physics and CFIF, Instituto Superior Tecnico, Universidade Tecnica de Lisboa, Avenue Rovisco Pais, 1049-001 Lisbon, Portugal
[f] ESRF, BP 220, 38043 Grenoble Cedex, France

ARTICLE INFO

Available online 3 December 2008

PACS:
73.21.−b
75.30.Jt
75.47.−m
73.23.Ra

Keywords:
Two-dimentional electron gas
Hall conductivity
Periodic magnetic field
Persistent currents

ABSTRACT

We study the energy spectrum and electronic properties of a two-dimensional (2D) spinless electron gas in a periodic magnetic field which has the symmetry of a triangular lattice. We show that the energy bands depend strongly on the value of the magnetic field. For large field the low-energy electrons are localized on closed rings where the magnetic field vanishes. This results in the appearance of persistent currents around these rings. We also calculate the intrinsic Hall conductivity, which is quantized when the Fermi level is in a gap.

1. Introduction

We have studied the dynamics of free electrons in a periodic magnetic field of zero average; such a periodic magnetic field could be created by a regular array of ferromagnetic nanocylinders [1]. In a previous paper [2], some of us have shown that anomalous Hall Effect (AHE) induced by spin chirality can be observed in a two-dimensional (2D) diluted magnetic semiconductor located on the top of this lattice of nanocylinders. In this paper we study a spinless electron gas submitted to the same periodic magnetic field; semi-classical consideration shows that in inhomogeneous magnetic field, electrons tend to localize near the lines of zero magnetic field [3]. The same tendency is found in the case of periodic magnetic field, leading to persistent currents as shown in the following. We also show that there is an intrinsic AHE in this system. This AHE is related to the non-trivial topology of the electron bands and does not require any uniform magnetization.

The model considered in this paper is a 2D electron gas in the x–y plane, while the periodic magnetic field is along the z-axis and can be written as

$$B(\mathbf{r}) = B_0\left[\cos\left(\frac{2\pi}{a}\mathbf{b}_1.\mathbf{r}\right) + \cos\left(\frac{2\pi}{a}\mathbf{b}_2.\mathbf{r}\right) + \cos\left(\frac{2\pi}{a}\mathbf{b}_3.\mathbf{r}\right)\right], \quad (1)$$

where the vectors $\mathbf{b}_i$ are the reciprocal lattice vectors of the triangular lattice.

The Hamiltonian is written as

$$H = \frac{\hbar^2}{2m}\left(-i\nabla - \frac{e}{\hbar c}\mathbf{A}(\mathbf{r})\right)^2, \quad (2)$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential related to $\mathbf{B}(\mathbf{r})$. In the Coulomb gauge, defined by the condition $\nabla\cdot\mathbf{A}(\mathbf{r}) = 0$, it is possible to choose $\mathbf{A}(\mathbf{r})$ periodic in space; thus the electrons are moving in a periodic potential created by $\mathbf{A}(\mathbf{r})$, and Bloch theorem can be applied to find the eigenfunctions of Hamiltonian (2).

Using Bloch theorem, the eigenfunctions of (1) can be written as

$$\psi_{n\mathbf{k}}(\mathbf{r}) = u_{n\mathbf{k}}(\mathbf{r})e^{i\mathbf{k}.\mathbf{r}}, \quad (3)$$

where n is a band index and $u_{n\mathbf{k}}(\mathbf{r})$ is periodic in space. Then $u_{n\mathbf{k}}(\mathbf{r})$ is an eigenfunction (associated with the energy $\varepsilon_{n\mathbf{k}}$) of the Hamiltonian $H_{\mathbf{k}}$:

$$H_{\mathbf{k}} = \frac{\hbar^2}{2m}\left(-i\nabla + \mathbf{k} - \frac{e}{\hbar c}\mathbf{A}(\mathbf{r})\right)^2. \quad (4)$$

More details on the calculation can be found in Ref. [4]. The shape of the energy bands and the wave functions depend on the dimensionless parameter $\alpha = -e(\sqrt{3}/4\pi)(ea^2/hc)B_0$.

* Corresponding author. Tel.: +33 476 88 10 98; fax: +33 476 88 11 91.
E-mail address: claudine.lacroix@grenoble.cnrs.fr (C. Lacroix).

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doi:10.1016/j.jmmm.2008.11.085

2. Electronic spectrum and wave functions

Fig. 1 shows the energy spectrum for several values of parameter α: for small α (Fig. 1a: $\alpha = 0.05$), electronic structure is close to a free-electron band structure, but gaps appear at the band crossing points. For large α (Fig. 1c: $\alpha = 5$), the bands are almost flat and well separated from each other, indicating that the electrons are localized in space. For intermediate values of α band crossings can occur at high symmetry points of the Brillouin Zone (Fig. 1b: $\alpha = 0.5$) and some gaps appear in the spectrum.

Fig. 2 shows the probability distribution of the Bloch states for two values of α: for small α, i.e. for small magnetic field, the electrons are rather delocalized over the unit cell (Fig. 2a, $\alpha = 0.05$); for larger magnetic field (Fig. 2b: $\alpha = 5$), the electrons are confined in the region where the magnetic field is close to zero: the lines $B(r) = 0$ are forming closed rings, as indicated on Fig. 2. This behavior corresponds to the semi-classical picture of electronic motion in a linear magnetic field [3]: the low-energy electrons are localized in regions where the energy of Landau level is minimum.

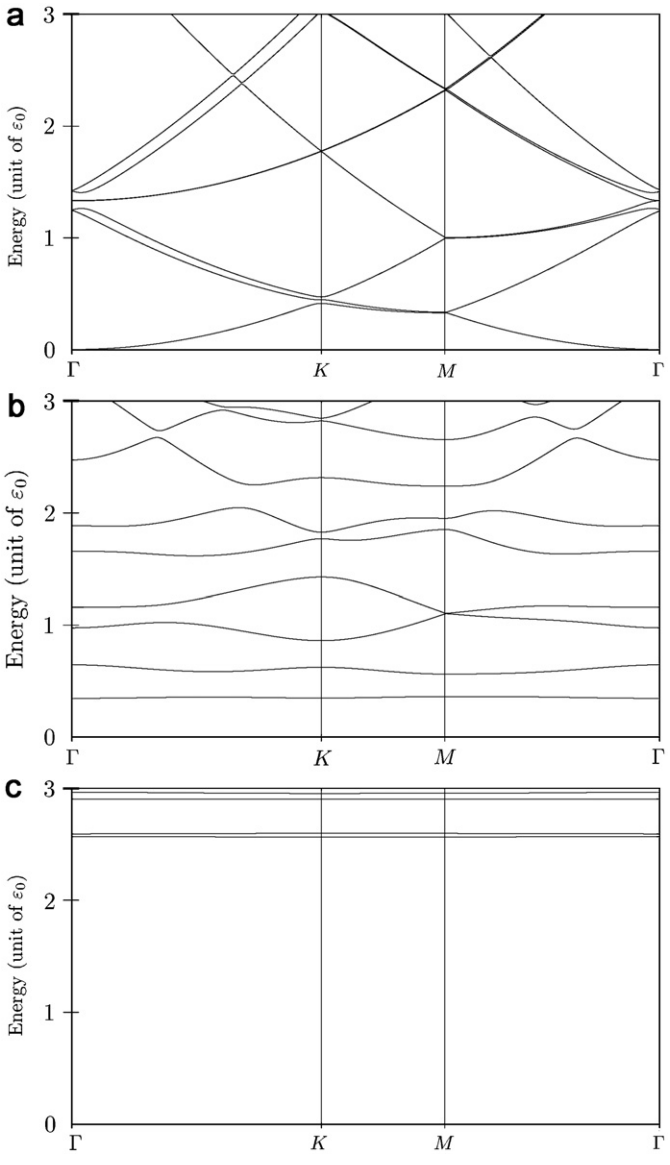

Fig. 1. Band structure for (a) $\alpha = 0.05$, (b) $\alpha = 0.5$, and (c) $\alpha = 5$.

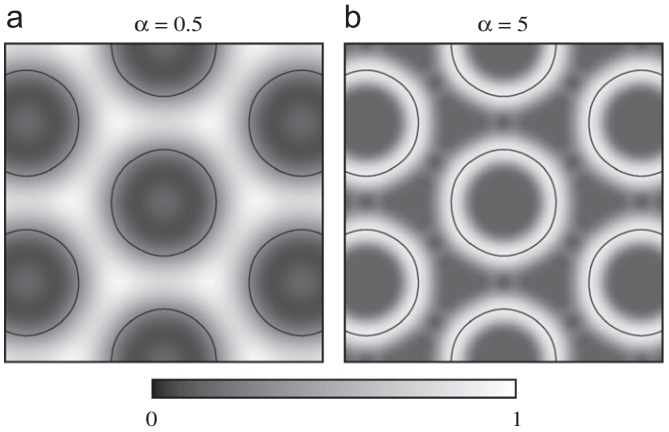

Fig. 2. Probability distribution for the Bloch state of the 1st band at Γ point for $\alpha = 0.5$ (a) and $\alpha = 5$ (b). The thick line is the line where the magnetic field $\mathbf{B}(\mathbf{r})$ vanishes.

3. Persistent currents

At large magnetic field, the electrons confined near the zero-field rings are creating a regular array of persistent currents; the local current density $\mathbf{J}(\mathbf{r})$ can be calculated from the wave functions $u_{n\mathbf{k}}(\mathbf{r})$:

$$J_{n\mathbf{k}}(\mathbf{r}) = \frac{\hbar}{2m}\mathrm{Re}\left[u^*_{n\mathbf{k}}(\mathbf{r})(-i\nabla + \mathbf{k} - \frac{e}{\hbar c}\mathbf{A}(\mathbf{r}))u_{n\mathbf{k}}(\mathbf{r})\right]. \quad (5)$$

For the Bloch state corresponding to Fig. 2b (Γ point of the 1st energy band), we found that the current density is non zero along the rings where $\mathbf{B}(\mathbf{r}) = 0$ [4]. Thus these Bloch states are forming a regular lattice of current contours in the 2D electron gas. The appearance of such persistent currents is due to the chirality of the electron motion in the non-uniform field.

This can be understood in a semi-classical picture: the electronic motion around the ring is 1D-like; the effective potential along these lines can be deduced from Eq. (4): it is not the same for electrons moving clockwise and anticlockwise as in the 1D case [3] (potential is different for k and $-k$); it results that the energy spectrum is asymmetric with respect to $k \to -k$, k being the electron momentum along the zero-field ring. Since the trajectories are closed lines, the spectrum is not only asymmetric, but also quantized: in the semi-classical picture, the quantized values of k are given by: $k_n - A_l = 2\pi n/L$, where A_l is the vector potential along the contour and L the length of the contour; this can also be written as $k_n = 2\pi(n+\phi/\phi_0)/L$, where ϕ_0 is the flux quantum and ϕ the flux of the field $\mathbf{B}_0$ over the unit cell.

Thus in this case there are two mechanisms leading to a persistent current: besides the usual Aharonov–Bohm effect (quantization of k), the chirality of the energy spectrum gives rise to a current even if ϕ/ϕ_0 is an integer.

4. Quantized Hall effect

In our model, the average magnetic field is zero, thus there is no ordinary Hall effect. The mechanism leading to AHE is similar to the intrinsic mechanism [5], i.e. it is related to the non-trivial topology of electron energy bands in the momentum space. Since there is no magnetization in this system it is different from the topological Hall effect proposed in Ref. [2].

Quantized Hall effect in a 2D tight-binding honeycomb lattice with periodic magnetic field has been studied by Haldane in Ref. [6]. In this model the AHE is related to the topology of the bands and the phase diagram has two phases corresponding to

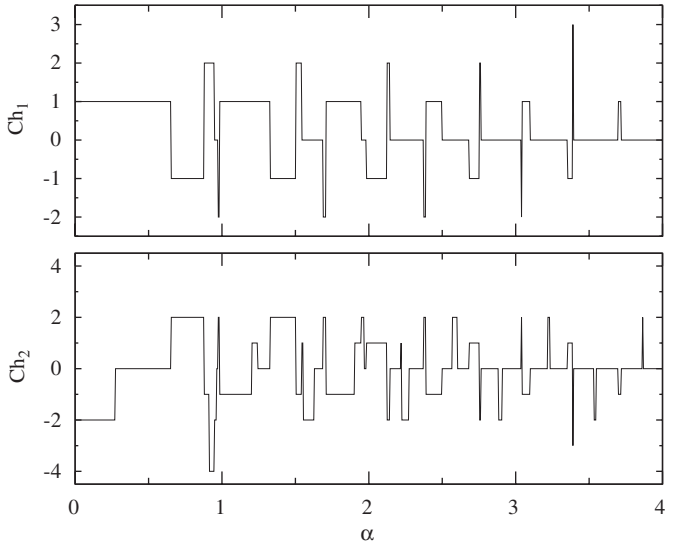

Fig. 3. Chern numbers of the 1st and 2nd energy bands, as function of α.

Chern numbers ± 1. In the present model the origin of AHE is similar, but it is more complex because the band structure is more complex.

Starting from the Kubo formula of conductivity, we can write [5]

$$\sigma_{xy} = \frac{e^2}{\hbar} \sum_{\mathrm{n}} \int \frac{d^2\mathbf{k}}{(2\pi)^2} f(\varepsilon_{\mathrm{n},\mathbf{k}}) \Omega_{\mathrm{n},\mathbf{k}}, \tag{6}$$

where $\Omega_{\mathrm{n},\mathbf{k}}$ is the Berry curvature of the Bloch state with wave vector $\mathbf{k}$ in the nth band, and $f(\varepsilon)$ the Fermi function. At zero temperature, and if the Fermi level is in a gap, expression (6) can be written as the sum of some integer, called Chern numbers $\mathrm{Ch_n}$, of all fully occupied bands. Each band is characterized by a different Chern number, positive or negative; consequently, if the Fermi level is in a gap, the Hall conductivity σ_{xy} is quantized. The value of the Chern numbers and of σ_{xy} changes when the gap between 2 occupied bands is closing.

We have calculated these Chern numbers for the 5 lowest energy bands. The results for the 1st and 2nd bands are shown on Fig. 3 as a function of the field: these numbers exhibit jumps when the value of the magnetic field is such that two bands cross. The mechanism is similar to the Haldane model [6], but here the behavior is more complex.

Hall conductivity can be deduced from these Chern numbers, if the Fermi level is in a gap. For example, if the total number of electrons is such that bands 1 and 2 are filled, σ_{xy} is equal to $\mathrm{Ch_1}$+$\mathrm{Ch_2}$ (in units of e^2/h); it is equal to -1 for small α, then jumps to +1 when increasing field. For large magnetic field all the Chern numbers tend to zero. For intermediate values of band filling, one has to sum the contribution of all filled bands and the non-integer contribution of the partially filled band.

5. Conclusions

We have shown that the band structure is controlled by the value of magnetic field. If this magnetic field is created by a regular array of ferromagnetic nanocylinders, the magnitude of the field was estimated in Ref. [2] to be of the order of 5 kG, close to the surface of the nanolattice and it can be varied by changing the distance between the 2D electron gas and the surface of the nanolattice. For a concrete realization of this system, the 2D electron gas can be a semiconductor layer with a lattice parameter a_0 much smaller than the lattice constant a of the periodic field. Impurity effects have been neglected in the calculation: this is justified if the electron mean free path is large compared to the characteristic sizes of the structure.

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