RELATIVISTIC THEORY OF SUPERCONDUCTIVITY

K. Capelle

Departamento de Química e Física Molecular
Instituto de Química de São Carlos, USP
13560-970 São Carlos, SP, Brazil

M.A.L. Marques

Departamento de Física Teórica
Faculdad de Ciencias, 47011 Valladolid, Spain

E.K.U. Gross

Institut für Theoretische Physik
Universität Würzburg, D-97074 Würzburg, Germany

1. RELATIVISTIC ORDER PARAMETERS

Relativistic effects in materials containing heavy atoms pervade many areas of condensed-matter physics [1-4]. The present chapter is devoted to the theory of relativistic effects in a particularly interesting class of materials, namely superconductors and other superfluids [5-9]. At its present stage that theory is limited to single-particle effects (on the level of the Dirac equation); a relativistic description of the particle-particle interaction is not attempted. However, already on the single-particle level several unexpected relativistic phenomena in superconductors emerge, among them a novel type of spin-orbit coupling, present in superconductors only. The origin and the physics of these phenomena are discussed in this contribution.

The fundamental ingredient in any description of superconductivity is the order parameter (OP). The BCS theory of superconductivity [10], in its original formulation, is not easily generalized to the relativistic domain. Effects of spatial inhomogeneity of the lattice and the OP, of different OP symmetries, and of magnetic fields are, however, easily incorporated within the framework of the Bogolubov-de Gennes (BdG) reformulation of the BCS theory [11], which also provides a suitable basis for the relativistic generalization. In the conventional BdG theory the OP describing the Cooper pairs can be written in terms of products of field operators, $\hat{\psi}_\uparrow(r)\hat{\psi}_\downarrow(r')$,

$\hat\psi_\downarrow(r)\hat\psi_\uparrow(r')$, $\hat\psi_\uparrow(r)\hat\psi_\uparrow(r')$, and $\hat\psi_\downarrow(r)\hat\psi_\downarrow(r')$, where the $\hat\psi_\sigma$ are components of the two-component Pauli spinor $\hat\Psi$. (All spatial arguments r are actually vectors $\vec r$, but for ease of notation the arrows are suppressed here and below.) Linear combinations of these products can be formed to represent the usual singlet and triplet states, which are even or odd under interchange of r and r', respectively [6,11-14]. The singlet pair is described, for example, by $\hat\Psi^T(r)i\hat\sigma_y\hat\Psi(r') = \hat\psi_\uparrow(r)\hat\psi_\downarrow(r') - \hat\psi_\downarrow(r)\hat\psi_\uparrow(r')$, which is manifestly even under interchange of r and r', and odd under that of spin up and spin down, as expected for a singlet state. Since the Pauli matrix $i\hat\sigma_y$ is the nonrelativistic time-reversal matrix, this way of writing the pair additionally brings out clearly that the BCS Cooper pair consists of time-conjugate single-particle states [11,13].

A convenient way of describing arbitrary pair states is by expanding the general 2×2 pair potential matrix $\hat\Delta(r,r')$ in the set of matrices $i\hat\sigma_y$ and $i\vec\sigma\hat\sigma_y$, where $\vec\sigma$ is the vector of Pauli matrices, and the three components of $i\vec\sigma\hat\sigma_y$ are the matrices appearing in the Balian-Werthamer parametrization of triplet OP [15],

$$\hat\Psi^T(r)\hat\Delta(r,r')\hat\Psi(r') = \Delta_0(r,r')\hat\Psi^T(r)i\hat\sigma_y\hat\Psi(r') + \vec\Delta(r,r')\hat\Psi^T(r)i\vec\sigma\hat\sigma_y\hat\Psi(r'). \quad (1)$$

Ordinarily one would use the unit matrix $\hat I$ and the vector of Pauli matrices $\vec\sigma$ as basis for this type of expansion, since by using this symmetry adapted set of matrices one achieves a separation of scalar quantities ($\hat\Psi^\dagger\hat I\hat\Psi$) from quantities transforming as a vector ($\hat\Psi^\dagger\vec\sigma\hat\Psi$), i.e, a classification with respect to irreducible representations (IR) of the rotation group. However, Cooper pairs are formed with two creation (or two annihilation) operators, and not with one creation and one annihilation operator, and a scalar-vector separation is only achieved in terms of the Balian-Werthamer matrices [15]. ($\hat\Psi^T\vec\sigma\hat\Psi$, for example, does not transform as a vector under rotations, while $\hat\Psi^T i\vec\sigma\hat\sigma_y\hat\Psi$ does.)

In a relativistic formulation the spin index σ is replaced by the label for the components of four-component Dirac spinors. Out of these four components one can form sixteen pairs, which can again be represented in terms of a complete set of matrices. One might be tempted to employ the sixteen 4×4 Dirac $\hat\gamma$ matrices for this purpose, since these usually lead to a separation into so called 'bilinear covariants', i.e., objects transforming under some IR of the Lorentz group $\mathcal{L}$ [16]. However, just as in the nonrelativistic case, the usual set of matrices does not achieve this if the object to be expanded is formed with two field operators of the same kind. A set of matrices in terms of which such a separation into IR of $\mathcal{L}$ is achieved, was derived in Ref. [6], where it was shown that the usual classification into five bilinear covariants, a Lorentz scalar (one component), a four vector (four components), a pseudo scalar (one component), an axial four vector (four components), and an antisymmetric tensor of rank two (six independent components), carries over to the superconducting OP if the Dirac $\hat\gamma$ matrices are replaced by a new set of matrices, denoted $\hat\eta$. Explicit expressions for these $\hat\eta$ matrices are given in Ref. [6]. These five OP, with a total of sixteen components, exhaust the possible pairings which can be formed from two Dirac spinors. As an example, consider the most important OP of all, the BCS singlet OP, which can be written in terms of Pauli spinors as $\hat\chi^{\mathrm{BCS,non-rel}} = \hat\Psi^T i\hat\sigma_y\hat\Psi$. Its relativistic generalization reads $\hat\chi^{\mathrm{BCS,rel}} = \hat\Psi^T\hat\eta_0\hat\Psi$, where $\hat\eta_0 = \hat\gamma^1\hat\gamma^3$ is one of the sixteen $\hat\eta$ matrices (the one yielding a Lorentz scalar), the $\hat\Psi$ are now four-component Dirac operators, and $\hat\gamma^1$ and $\hat\gamma^3$ are two of the standard Dirac matrices.

Initially [5], the relativistic generalization of the BCS singlet OP was constructed by replacing the nonrelativistic time-reversal matrix $\hat{t} = i\hat{\sigma}_y$ in $\hat{\chi}^{\mathrm{BCS,non-rel}}$ by the relativistic one, $\hat{T} = \hat{\gamma}^1\hat{\gamma}^3$, when constructing $\hat{\chi}^{\mathrm{BCS,rel}}$, i.e., it was postulated that relativistic Cooper pairs still consist of time-conjugate states. In later work [6], this turned out to be unnecessary, since the matrix $\hat{\eta}_0 \equiv \hat{T}$ automatically emerges as that of the sixteen $\hat{\eta}$ matrices leading to covariant pairs, which reduces to the BCS singlet OP in the nonrelativistic limit. It is important to point out that the relativistic generalization of the BCS OP is uniquely determined by both prescriptions, and that both indeed lead to the same result, $\hat{\chi}^{\mathrm{BCS,rel}} = \hat{\Psi}^T\hat{\eta}_0\hat{\Psi}$.

2. DIRAC EQUATION FOR SUPERCONDUCTORS

In terms of the relativistic OP one can now generalize the entire theory of superconductivity to the relativistic domain. At the heart of this generalization are the Dirac-Bogolubov-de Gennes equations (DBdGE), which for a BCS-like spin-singlet superconductor read [5,6]

$$\hat{\gamma}^0[c\vec{\gamma}\cdot\vec{p} + mc^2(1-\hat{\gamma}^0) + q\gamma^\mu A_\mu]u_n(r) + \int d^3r'\Delta(r,r')\hat{\eta}_0 v_n(r') = E_n u_n(r) \qquad (2)$$

$$-\hat{\gamma}^0[c\vec{\gamma}\cdot\vec{p} + mc^2(1-\hat{\gamma}^0) + q\gamma^\mu A_\mu]^* v_n(r) - \int d^3r'\Delta^*(r,r')\hat{\eta}_0 u_n(r') = E_n v_n(r), \quad (3)$$

where both u_n and v_n are four-component (Dirac) spinors, representing particle and hole amplitudes, while γ^μ is the four vector of $\hat{\gamma}$ matrices (in standard notation, and with a summation over repeated greek indices implied), and $\vec{\gamma}$ the corresponding three vector, containing $\hat{\gamma}^1$,$\hat{\gamma}^2$, and $\hat{\gamma}^3$. A_μ is the four potential, and Δ the pair potential. Equations (2) and (3) generalize the conventional Bogolubov-de Gennes equations (BdGE) for spin-singlet superconductors [11] to the relativistic domain [5,6] in the same way in which the conventional Dirac equation generalizes Schrödinger's equation [3,16]. The potentials $\Delta(r,r')$ and $A_\mu(r)$ in Eqs. (2) and (3) are to be regarded as effective potentials incorporating the electron-electron interaction in the spirit of mean-field [11] or density-functional [12] theory.

The pairing of time-conjugate pairs, described by the matrix $\hat{\eta}_0$, is, in fact, characteristic only of BCS-like spin-singlet superconductors. For triplet superconductors, even in the nonrelativistic case, one needs parity conjugation $\hat{p}$ in addition to time reversal $\hat{t}$, in order to express the order parameter and the underlying pairing state in terms of fundamental discrete symmetries. From the two operations, $\hat{p}$ and $\hat{t}$, acting on two-component (Pauli) spinors one can form four antisymmetric pairing states, representing singlets and triplets [6,13]. The relativistic generalization, $\hat{P}$ and $\hat{T}$, of the symmetry operations $\hat{p}$ and $\hat{t}$ is, of course, well known [3,16] and the above nonrelativistic prescription for forming pair states is easily generalized to the relativistic case [6]. Apart from $\hat{P}$ and $\hat{T}$, relativity also provides a third fundamental discrete symmetry, charge conjugation $\hat{C}$, which can, as a matter of principle, also be used to form pair states. From the three operations $\hat{P}$, $\hat{T}$, and $\hat{C}$, acting on four-component Dirac spinors, one can form sixteen antisymmetric pair states, which are nothing

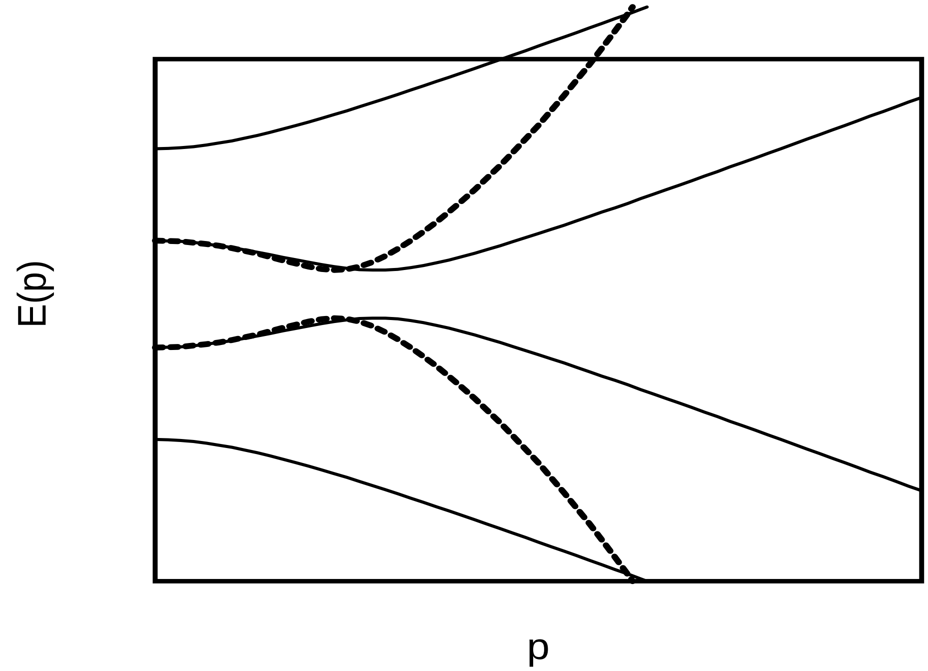

Figure 1. *Solid lines:* Schematic energy spectrum for a relativistic BCS-like superconductor with a point-contact pair potential, in a constant external potential (i.e., disregarding lattice and band structure effects), calculated by analytical diagonalization of the DBdGE (2) and (3). In order to make the superconducting gap and the relativistic gap visible on the same scale, Δ_0 and μ were artificially enlarged. *Dashed lines:* Conventional nonrelativistic BCS energy spectrum for the same values of Δ_0 and μ. More details are given in the main text.

but the sixteen combinations entering the five superconducting bilinear covariants, obtained independently above, from considerations of Lorentz covariance.

The Dirac-Bogolubov-de Gennes equations incorporating all these OP read [17]

$$\hat{\gamma}^0[c\vec{\gamma}\cdot\vec{p}+mc^2(1-\hat{\gamma}^0)+q\gamma^\mu A_\mu]u_n(r)-\sum_{i=0}^{15}\int d^3r'\Delta_i(r,r')\hat{\eta}_i^\dagger v_n(r')=E_n u_n(r) \quad (4)$$

$$-\hat{\gamma}^0[c\vec{\gamma}\cdot\vec{p}+mc^2(1-\hat{\gamma}^0)+q\gamma^\mu A_\mu]^* v_n(r)+\sum_{i=0}^{15}\int d^3r'\Delta_i^*(r,r')\hat{\eta}_i^T v_n(r')=E_n v_n(r), \quad (5)$$

where the sums are over all sixteen matrices $\hat{\eta}_i$, and the corresponding pair potentials Δ_i. These equations thus contain all five possible OP compatible with Lorentz covariance. In practice one expects any given interaction leading to superconductivity to produce only one of these covariant OP. The DBdGE for each of these are easily obtained from Eqs. (4) and (5) by setting all pair potentials Δ_i, except for those belonging to the IR under consideration, equal to zero. (In the case of the Lorentz scalar one recovers Eqs. (2) and (3) in this way.)

Eqs. (2) and (3), can be diagonalized analytically [5] for spatially uniform superconductors, employing a point-contact pair potential $\Delta(r,r')=\Delta_0\delta(r-r')$ and setting $A_\mu = (v, 0, 0, 0)$, where Δ_0 and v are constant in space. The resulting energy spectrum is displayed in Fig. 1, together with its nonrelativistic limit, the conventional BCS spectrum for the same choice of $\Delta(r, r')$ and μ. For small momenta and energies the relativistic and nonrelativistic spectra are very similar, but the relativistic superconducting gap is slightly shifted to higher momenta with respect to the nonrelativistic one. An analytical expression for this shift was obtained in Ref. [5]. For larger momenta and energies the relativistic spectrum aquires features of a free particle solution of the Dirac equation, such as a linear dispersion relation.

3. PAULI EQUATION FOR SUPERCONDUCTORS

Although the task to set up a relativistic generalization of the Bogolubov-de Gennes formulation of the BCS theory is formally completed with the derivation of the Dirac-Bogolubov-de Gennes equations and all possible order parameters they may contain, the underlying physics comes out much clearer by leaving the level of the Dirac equation and proceeding to the weakly relativistic and the nonrelativistic limits. These limits are important because (i) the nonrelativistic limit of the DBdGE must be the conventional BdGE, hence recovering this limit constitutes an essential test of the theory, and (ii) in the weakly relativistic limit (defined as including terms up to second order in v/c, where v is a typical particle velocity) one obtains the first relativistic corrections to the conventional theory of superconductivity.

By systematic elimination of the lower components of the DBdGE (i.e., those which are suppressed by factors of v/c in the weakly relativistic limit) one obtains from Eqs. (2) and (3) the pair of equations [5,7]

$$[h(r) + \delta h(r)]\, u_n(r) + \int d^3r' \, [\Delta(r, r') i\hat{\sigma}_y + \delta\Delta(r, r')]\, v_n(r') = E_n u_n(r) \qquad (6)$$

$$-\, [h(r) + \delta h(r)]^*\, v_n(r) - \int d^3r' \left[\Delta^*(r', r) i\hat{\sigma}_y - \delta\Delta^\dagger(r', r)\right] u_n(r') = E_n v_n(r) \qquad (7)$$

where u_n and v_n are two-component (Pauli) spinors, $h(r) = \frac{1}{2m}[\vec{p} - \frac{q}{c}\vec{A}(r)]^2 + v(r) - \mu\vec{\sigma}\vec{B}(r)$ is the normal-state Hamiltonian, including the vector potential $\vec{A}$ and the magnetic field $\vec{B}$, and $\mu = \hbar q/(2mc)$ is the Bohr magneton. Neglecting the relativistic correction terms δh and $\delta\Delta$, one obtains from Eqs. (6) and (7) the traditional BdGE for spin-singlet superconductors [11]. The term

$$\delta h(r) = \frac{1}{4m^2c^2}\left[\hbar\vec{\sigma}\cdot[\nabla v(r)] \times \vec{p} + \frac{\hbar^2}{2}\nabla^2 v(r) - \frac{p^4}{2m}\right] \qquad (8)$$

represents the second-order relativistic corrections appearing already in the normal state, i.e., spin-orbit coupling, which involves the gradient of the lattice potential, the Darwin term, containing second derivatives of the lattice potential, and the mass-velocity correction, which is quartic in p. Similarly,

$$\delta\Delta(r, r') = \frac{1}{4m^2c^2}\left[\hbar\vec{\sigma}\cdot[\nabla\Delta(r, r')] \times \vec{p}' + \frac{\hbar^2}{2}\left(\nabla + \nabla'\right)^2 \Delta(r, r')\right] i\hat{\sigma}_y \qquad (9)$$

contains the second-order relativistic corrections involving the pair potential, i.e., those which appear only in the superconducting state. (The prime on p and ∇ denotes a derivative with repect to the primed coordinate.) These terms depend on the pair potential in a similar way in which those of δh depend on the lattice potential, and are referred to as the *anomalous* spin-orbit coupling (ASOC) and anomalous Darwin terms, respectively.

More correction terms arise if additionally triplet OP are included. In this case already in the nonrelativistic (zero order in $1/c$) terms in Eqs. (5) and (6) one must add to the term $\Delta(r,r')i\hat{\sigma}_y$ the sum $\sum_{j=x,y,z}\Delta_j(r,r')\hat{\sigma}_j\hat{\sigma}_y$ [14,15]. Furthermore, Eqs. (8) and (9) stem from Eqs. (2) and (3), and hence do not include OP involving charge conjugation, neither for singlets nor for triplets. The generalization of Eqs. (8) and (9) involving *all* possible OP discussed in the previous sections is found from Eqs. (4) and (5) [17] and reads

$$\delta h(r) = \frac{1}{4m^2c^2}\left[\frac{\hbar^2}{2}\nabla^2 v(r) + \hbar\vec{\sigma}\cdot(\nabla v)\times\vec{p} - \frac{p^4}{2m} + \frac{1}{2mc^2}\hat{\mathcal{D}}_{12}\hat{\mathcal{D}}_{21}^*\right] \tag{10}$$

and

$$\delta\Delta(r,r') = \frac{1}{2mc}\left[\vec{\sigma}\left(\vec{p}-\frac{q}{c}\vec{A}(r)\right)\hat{\mathcal{D}}_{21} + \hat{\mathcal{D}}_{12}\vec{\sigma}^*\left(\vec{p}'-\frac{q}{c}\vec{A}(r')\right)^*\right]$$
$$+\frac{1}{4m^2c^2}\left[\vec{\sigma}\vec{p}\hat{\mathcal{D}}_{22}\vec{\sigma}^*\vec{p}'^* + \frac{\hbar^2}{2}\left(\nabla^2\hat{\mathcal{D}}_{11} + \hat{\mathcal{D}}_{11}\nabla'^2\right)\right], \tag{11}$$

where the 4×4 matrix $\hat{\mathcal{D}}$ is defined in terms of the complete set of $\hat{\eta}$ matrices as

$$\hat{\mathcal{D}} = -\sum_{i=0}^{15}\Delta_i(r',r)\eta_i^\dagger, \tag{12}$$

and the action of the product $\hat{\mathcal{D}}_{12}\hat{\mathcal{D}}_{21}^*$ on a function $f(r)$ should be interpreted as

$$\hat{\mathcal{D}}_{12}\hat{\mathcal{D}}_{21}^* f(r) = \sum_{i,j=0}^{15}\int dr''\Delta_j(r'',r)\Delta_i^*(r',r'')f(r')\eta_{j12}^\dagger\eta_{i21}^T. \tag{13}$$

The subscripts in, e.g., $\hat{\mathcal{D}}_{12}$, refer to 2×2 submatrices, i.e., $\hat{\mathcal{D}}_{12}$ is the upper right block of $\hat{\mathcal{D}}$. If all pair potentials but that multiplying $\hat{\eta}_0$ are set equal to zero (i.e., if one specializes to a BCS-like spin-singlet OP) these equations reduce to (8) and (9). On the other hand, through the OP matrices $\hat{\eta}_1\ldots\hat{\eta}_{15}$ and the corresponding pair potentials they contain all relativistic corrections for singlet and triplet superconductors up to second order in $1/c$, including those forming their Cooper pairs via charge conjugation.

We now turn to a discussion of the physics behind the weakly relativistic correction terms of Eqs. (8), (9), (10), and (11). We start by considering only the corrections appearing for BCS-like singlet superconductors, i.e., those given in Eqs. (8) and (9). The spin-orbit, mass-velocity, and Darwin corrections to $h(r)$, contained in Eq. (8), appear already in the normal state and are well known from other areas of physics [3,4,16]. Although these corrections are thus not of superconducting

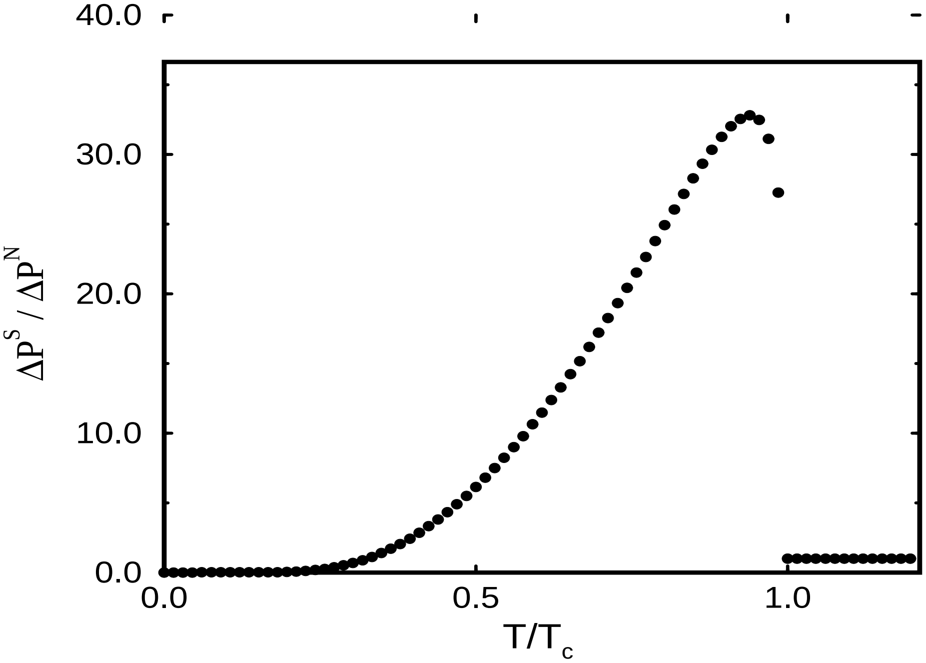

Figure 2. Difference in power absorption ΔP^S between left-hand circularly polarized light and right-hand circularly polarized light in the superconducting state, divided by the corresponding normal-state difference ΔP^N, as a function of temperature divided by the transition temperature T_c. The difference ΔP^S is proportional to $\Im[\hat{\sigma}_{xy}^S(\omega, T, H)]$, the imaginary part of the offdiagonal elements of the conductivity tensor in the superconductor, as a function of frequency, temperature and magnetic field ($\omega = 4.5meV$ and $H = 0.05T$ for the data in the figure) [19].

origin, their effect on observables in a superconductor can be dramatically modified by superconducting coherence. The conventional spin-orbit coupling (SOC) term in Eq. (8), for example, breaks rotational symmetry of the spin degrees of freedom with respect to the orbital degrees of freedom. It is well known that this broken symmetry gives rise to circular dichroism in the magneto-optical response of metals [4]. Below the critical temperature T_c the dichroic response of a superconductor is drastically modified. A microscopic theory of this modification was recently worked out in Refs. [18] and [19] on the basis of Eqs. (6) to (9), and the results can qualitatively account for experimental observations. As a representative result we present in Fig. 2 a graph displaying the circular dichroic response of a simple model superconductor as a function of temperature, normalized by dividing by the corresponding normal-state result [19]. Without SOC both numerator and denominator would be zero, while without superconducting coherence the curve would be flat throughout. The strong peak seen immediately below the transition temperature thus arises only from the simultaneous presence of superconducting coherence and SOC [19].

The mass-velocity term, on the other hand, provides a small correction to the bare electron mass. It attests to the ingenuity of experimental physicists that the very small change of the Cooper pair mass in a superconductor due to the relativistic mass enhancement has already been measured [20,21,22]. Another consequence of this term in superconductors is the small shift of the energy gap of a homogeneous superconductor, mentioned at the end of Section 2.

The anomalous spin-orbit and Darwin corrections to $\Delta(r,r')$, given in Eq. (9), are fundamentally different from those to $\delta h(r)$, since they depend explicitly on the pair potential and are nonzero only in the superconducting state of matter. These terms were derived for the first time in Ref. [5]. The anomalous spin-orbit coupling provides a contribution to dichroism in superconductors, which can be distinguished from that of conventional SOC due to their very different temperature dependence [18,19]. The first appearence of an SOC term containing the pair potential, i.e., the term nowadays called ASOC, dates back to 1985, when Ueda and Rice postulated such a term on group theoretical grounds in their phenomenological treatment of p-wave superconductivity [23]. However, at that time it was not clear how such a term could be obtained microscopically, and what its detailed form was. These questions were answered only ten years later on the basis of the theory outlined above [5]. Both Darwin terms, the conventional and the anomalous one, have also been rederived phenomenologically [9]. This rederivation showed that the anomalous Darwin term can be understood as a consequence of relativistic fluctuations of paired particles in the pair potential of the superconductor, in a similar way in which the conventional Darwin term can be understood as a consequence of fluctuations of charged particles in the electric (lattice) potential [16].

We now turn to the relativistic corrections appearing for more exotic superconductors, contained in Eqs. (10) and (11). A closer look at these equations reveals that there are several terms which are straightforward generalizations of those in Eqs. (8) and (9), but also two terms which do not have counterparts in those equations. The first of these is the term containing the pair potential operator in $\delta h(r)$. It is a new feature of these equations, not present in other Bogolubov-de Gennes equations, that terms related to superconductivity appear also on the diagonal of the BdG matrix. The second is the cross term in $\delta\Delta(r,r')$, containing both the pair potential operator and the vector potential. Although this latter term has the usual minimal coupling form (in which $\vec{p}$ is replaced by $\vec{p}-(q/c)\vec{A}$) it is, in fact, highly unusual, since it leads to a term containing the pair potential multiplied by the vector potential. It is of considerable fundamental interest that such unusual coupling becomes possible in the theory of superconductivity, but detailed consequences of this remain to be explored. Both of these novel terms contain the offdiagonal blocks of the matrix $\hat{\mathcal{D}}$, which are nonzero only for OP formed with relativistic charge conjugation as fundamental discrete symmetry. These OP do vanish in the strictly nonrelativistic limit [6,7,17], and are thus intrinsically relativistic. Interestingly, for these OP there are no anomalous Darwin and spin-orbit terms, since these involve only the diagonal blocks of the matrix $\hat{\mathcal{D}}$, which are zero for the OP involving charge conjugation. In the absence of this purely relativistic type of pairing the matrix $\hat{\mathcal{D}}$ is block diagonal, and all terms containing $\hat{\mathcal{D}}_{12}$ and $\hat{\mathcal{D}}_{21}$ vanish. The remaining terms provide the leading order relativistic corrections to the nonrelativistic singlet [11] and triplet [14] Bogolubov-de Gennes equations.

To conclude this section we emphasize that all results up to this point are independent of the form of the interaction leading to superconductivity. The role of the interaction is to select among the *possible* OP, discussed in Section 1, the one which is *realized* in a given superconductor. Relativity alone does not determine what interactions can give rise to superconductivity, but it puts strong constraints on the form and transformation behaviour of the resulting OP (e.g., it limits their number to sixteen, forming the five covariant combinations listed above). Furthermore, regardless of the detailed nature of the interaction, if there is superconductivity at all, then there will be relativistic corrections to it, and the form of these is universal, i.e, independent of the interaction itself. For example, any superconductor with a BCS-like singlet OP, no matter what (quasi)particles are paired and by what interaction, must display the relativistic corrections contained in Eqs. (8) and (9).

4. EXPERIMENTAL ASPECTS AND OUTLOOK

By now there are several predictions for experiment, which have been extracted from the general theory outlined above. These are (i) the existence of new types of order parameters in a relativistic theory, which vanish in the nonrelativistic limit [6,7,17], (ii) a small shift in the position of the energy gap [5], (iii) relativistic fluctuations in the pair potential, giving rise to the anomalous Darwin term [9], (iv) a drastic modification of the contribution of SOC to the dichroic response of superconductors below T_c [18,19], (v) a contribution of ASOC to the same dichroic response [18,19], (vi) the influence of relativity on the OP symmetry, even for conventional superconductors [5,6,7], and (vii) the relativistic change of the Cooper pair mass [5,7,20,21,22]. Clearly, the task to develop the theory to the point at which meaningful comparison with experiment can be made is not an easy one, and only first (preliminary, but encouraging) steps could be reported in this chapter. However, a few general considerations may facilitate experimental detection of these effects:

First of all, since relativistic effects become important at high velocities, one should study superconductors in which the electrons move rapidly. This is the case either if the normal-state metal has a very high Fermi velocity, or if there are sufficiently heavy elements in the lattice, whose orbitals are hybridized with the conduction band. Examples of such systems are the heavy-fermion superconductors like UPt_3 – containing some of the heaviest elements found in solid-state compounds, the high-temperature superconductors – which include between the CuO layers heavy elements like Ba, La or Hg, or BCS superconductors with heavy atoms in the lattice, such as Pb. Apart from high velocities, rapid spatial variations of the pair potential particularly favour the anomalous relativistic effects (ASOC and the anomalous Darwin term), since these depend on derivatives of the pair potential. Situations in which the pair potential varies rapidly are, e.g., superconductivity in thin films and small grains, multilayers, and the vortex state. In all these cases a short coherence length allows more rapid spatial variations, and thus increases the anomalous relativistic effects (an observation which points again at high-temperature superconductors). Presumably, the most important relativistic corrections in these materials are those of Eq. (8) (which are already routinely taken into account in relativistic band-structure codes for the normal state, but not included in most approaches to superconductivity). However, new and interesting phenomena can be expected to arise as a consequence of the anomalous corrections of Eq. (9).

A more detailed exploration of the fascinating territory of the interplay of superconducting coherence and relativistic covariance remains a task for the future, but already now it seems certain that the two areas can benefit from each other both in the study of down-to-earth superconductors, which was the subject of this paper, and in the field of cosmological and high-energy analogies of coherence and superconductivity [24], to which many of our results also apply.

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