Feigenbaum scenario in the dynamics of a metal–oxide semiconductor heterostructure under harmonic perturbation.
Golden mean criticality

C.P. Cristescu a, B. Mereu b,1, Cristina Stan a,*, M. Agop c

a Department of Physics I, “Politehnica” University of Bucharest, Bucharest 060042, Romania
b Max Planck Institute of Microstructure Physics, Weinberg 2, D-06120 Halle, Germany
c Department of Physics, “Gh. Asachi” Technical University of Iassy, Blvd. Mangeron, 700029 Iassy, Romania

Accepted 23 August 2007

Abstract

Experimental investigations and theoretical analysis on the dynamics of a metal–oxide semiconductor heterostructure used as nonlinear capacity in a series RLC electric circuit are presented. A harmonic voltage perturbation can induce various nonlinear behaviours, particularly evolution to chaos by period doubling and torus destabilization. In this work we focus on the change in dynamics induced by a sinusoidal driving with constant frequency and variable amplitude. Theoretical treatment based on the microscopic mechanisms involved led us to a dynamic system with a piecewise behaviour. Consequently, a model consisting of a nonlinear oscillator described by a piecewise second order ordinary differential equation is proposed. This kind of treatment is required by the asymmetry in the behaviour of the metal–oxide semiconductor with respect to the polarization of the perturbing voltage. The dynamics of the theoretical model is in good agreement with the experimental results. A connection with El Naschie’s $E$-infinity space-time is established based on the interpretation of our experimental results as evidence of the importance of the golden mean criticality in the microscopic world.

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1. Introduction

The chaos in nonlinear systems has been extensively studied in various interdisciplinary fields. Numerical simulation and experimental methods of chaos dynamics were reported in physical systems with major impact on many aspects of science (high energy physics [1,2], plasma physics [3], conscious information processing [4]) and applications as in chemistry [5], neuroscience [6] and engineering [7].

The behaviour of a nonlinear oscillator consists in a diversity of dynamical phenomena when the nonlinearity is changed. Since a series RLC circuit with nonlinear capacity or resistance is a typical oscillator for understanding the
phenomena in more complicated systems, many theoretical as well as experimental studies have been dedicated to this model [8,9]. A review of this kind of research with the nonlinearity originating in semiconductor structures can be found in [10–12].

In this paper, experimental investigations and theoretical analysis on the Feigenbaum scenario of evolution to chaos [13] in the dynamics of a particular set up consisting in a series RLC electric circuit where the nonlinear capacity is a Si based metal–oxide semiconductor heterostructure (MOSH) are presented. We focus on the change in the dynamics of this experimental arrangement induced by a sinusoidal perturbation with constant frequency and variable amplitude. In the dynamics of this particular structure, various scenarios of evolution to chaos as torus destabilization scenario and Feigenbaum scenario (period doubling) were observed. We also found this structure to present other types of nonlinear behaviour, particularly autonomous stochastic resonance [14].

A microscopic analysis of the dynamics of MOSH under periodic excitation is considered and the result is used in the numerical simulation.

The deep implication of the golden mean in our experimental and theoretical data suggests the importance of the golden mean criticality in the microscopic dynamics theory and a connection with El Naschie’s $E$-infinity space-time.

2. Experimental results and theoretical considerations

The experimental set-up is shown in Fig. 1. It consists of an RLC series circuit with the MOSH acting as nonlinear capacitance, in series with a linear capacitance $C_0$, a resistance $R$ and an inductance $L$. The MOSH was fabricated by creation of an oxide film on a p-type Si (100) substrate with a resistivity of 1–10 $\Omega$ cm. The thickness ($286 \pm 15$ nm) of the film has been determined by scanning electron microscopy (SEM). X-Ray-diffraction (XRD) analysis showed a polycrystalline structure for the oxide layer. The electrical contacts have been realised by vacuum evaporation of Al. The top contact (on the oxide film) was profiled using a mask, while the bottom one (on the Si substrate) was produced by Al evaporation on the entire surface.

The RLC circuit is externally driven by a harmonic voltage generator (HP8116A). The voltage drop on the resistance (proportional to the derivative of the dielectric displacement $D$) and the voltage drop on the linear capacitor (proportional to $D$) have been simultaneously monitored and acquired on a PC with a sampling rate of 10 MHz.

The results for particular values of the parameters indicated on each graph are shown in Fig. 2. On the left are phase portraits (resistor voltage versus linear capacitor voltage) and on the right, the corresponding power spectra for the voltage drop on the linear capacitor. With increasing amplitude, a clear period doubling evolution to chaos (the Feigenbaum scenario) is observed.

We begin the theoretical analysis considering the equation for the voltage in an RLC series circuit with a standard capacitor $C_0$ and a metal–oxide semiconductor capacitor under ac voltage driving:

$$LD''(t) + RD'(t) + D(t)/C_0 + V_{MOS}(t) = V_0 \sin(\omega t)$$

where $D$ is the electric induction, proportional with the charge on the capacitor plate and

$$V_{MOS}(t) = V_{FB} + V_{ox}(t) + \psi_S(t),$$

$V_0$ is the amplitude of the ac driving signal, $\omega$ is its angular frequency, $L$ and $R$ are the series inductance and resistance, respectively, $V_{ox}$ is the voltage on the gate dielectric, $V_{FB}$ is the flat-band voltage and $\psi_S$ is the potential at the silicon surface. The surface silicon potential depends on the regime at the silicon surface. The substrate is p-type and, consid-

![Fig. 1. Experimental set-up.](image-url)
Fig. 2. Experimental graphs: on the left-phase portraits (resistor voltage versus linear capacitor voltage) and on the right the corresponding power spectra.

Considering that the driving signal is high amplitude and high frequency, in the positive cycle of a complete period the only regime is depletion. At frequencies over 1 kHz and room temperature, minority carriers have a relatively slow response and the inversion layer is not formed.
In the depletion approximation the surface silicon potential is:

$$\psi_S^{(+)} = \frac{Q^2}{2\varepsilon_S qN_A}.$$  

(3)

Here, $N_A$ is the silicon doping, considered uniform at the surface, $q$ is the elementary charge, $\varepsilon_S$ is the dielectric constant of silicon, $Q = qN_AW$ is the ionic charge in the depletion region, whose depth is $w$. $Q$ is in fact the electrode charge for the MOS capacitor and equals the variable $D$ in (1).

For the negative cycle of the period, the silicon surface is in strong accumulation. The electric field $F(x)$ close to the silicon surface is expressed in the frame of the exact theory as:

$$F_S^{(-2)} = \frac{\varepsilon_S \beta E_F}{2q} \exp \left( \beta \psi_S^{(-1)} \right).$$

(5)

Here the sign convention for $D = \varepsilon_S F_S^{(-2)}$ is as follows: positive $D$ is defined when the net charge at the Si surface is positive. The surface potential deduced from (5) is:

$$\psi_S^{(-1)} \approx \frac{1}{\beta} \ln \left( 1 + \frac{\varepsilon_S \beta E_F^{(-2)}}{2q} \right) = \frac{1}{\beta} \ln \left( 1 + \frac{\beta D^2}{2q \varepsilon_S \exp(\beta E_F)} \right).$$

(6)

Returning to (1), we observe that this becomes now a piecewise second order ordinary differential equation:

$$LD''(t) + RD'(t) + D \left( \frac{1}{C_0} + \frac{1}{C_{ox}} \right) \left\{ \begin{array}{ll} \frac{\beta^2}{2q \varepsilon_S N_A}, & \text{for } D \leq 0 \\ \frac{1}{\beta} \ln \left( 1 + \frac{\beta D^2}{2q \varepsilon_S \exp(\beta E_F)} \right), & \text{for } D > 0 \end{array} \right\} = V_0 \sin(\omega t) - V_{FB}.$$  

(7)

3. Numerical results and conclusions

With proper notation, (7) is cast in the more convenient form:

$$ax'' + bx' + cx^3 + \begin{cases} mx^2, x \leq 0 \\ n \ln(1 + dx^2), x > 0 \end{cases} = q \sin 2\pi gt - h.$$  

(8)

We integrate this system using the Dormand-Prince 8(5,3) algorithm [15]. The results are shown in Fig. 3. On the left, the phase portraits and on the right, the corresponding spectra for appropriate values of the amplitude (shown on each graph) and for a frequency of the harmonic excitation $g = 10/2\pi$ are presented. We observe good qualitative agreement with the experimental results shown in Fig. 2.

Our investigation, both experimental and theoretical demonstrates that the Feigenbaum scenario of evolution to chaos for the MOSH system involves clear microscopic origins.

Using the critical values of the experimental control parameter $V_0$ (the amplitude of the ac driving signal) for the first three bifurcations (shown in Fig. 2), we obtain an estimate of the Feigenbaum [13] constant characteristic of our bifurcation sequence:

$$\frac{8.35 - 7.46}{8.69 - 8.35} = 0.89 \approx 2.6176 \approx \frac{1}{\phi^2}. $$

(9)

We observe that this can be remarkably well expressed in terms of the golden mean $\phi = (\sqrt{5} - 1)/2$. Assuming geometric convergence and using this value as true Feigenbaum constant, the computation of the value of the control parameter for which the chaotic dynamics sets in gives:

$$\frac{8.35 - 7.46}{8.69 - 8.35} = 0.89 \approx 2.6176 \approx \frac{1}{\phi^2}. $$

(9)
Fig. 3. Numerical simulations: on the left-phase portraits \((x_{n+20}, x_n)\) and on the right the corresponding power spectra. Additionally, we used: \(a = 2.8; \ b = -10; \ c = 5; \ h = 2; \ m = 10, n = 4, d = 1.\)

\[
\frac{8.35 - 7.46}{1/\phi^2 - 1} + 8.35 = 8.90
\]

which is in excellent agreement with the experimental value, 8.90.
The same computation, performed on the results of the numerical simulation gives:
\[
\frac{153 - 147.8}{155 - 153} = \frac{5.2}{2} \approx 2.60,
\]
in good agreement with the experimental result in (9).

Using the recurrence rule, we can compute the following bifurcation values and the limit of the series. The results are again in good agreement with the values obtained from the bifurcation diagram. For example, the theoretical limit value is:
\[
\frac{153 - 147.8}{1/\phi^2 - 1} + 153 \approx 156.2,
\]
which is exactly the same value as shown in Fig. 3, the bottom spectrum.

Recently, the Feigenbaum scenario and the golden mean renormalization group mathematical formalism were closely related to the \(E\)-infinity space-time theory of El Naschie [1,2,16–18]. Important reasons to support this relationship are [2]:

(i) In any space-time theory applicable to quantum physics in the sense of Minkowski–Einstein geometrization, we must have a turbulent vacuum or ground state. This turbulence is referred to more conventionally as vacuum fluctuation and the hope is that deterministic chaos can model it.

(ii) The geometry of chaos is fractals and Cantor’s triadic set is probably the simplest fractal ever conceived.

The presence of the Feigenbaum scenario, with the Feigenbaum constant clearly expressible in terms of the golden mean, suggests a connexion with the El Naschie \(E\)-infinity space-time theory [1,2,17,18], where the golden mean \(\phi = (\sqrt{5} - 1)/2\) is the fundamental entity. Our experimental results can be considered as evidence for the important implication of the golden mean constant in the microscopic world.

References