Unsteady-state lock-in thermography – Application to shunts in solar cells

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ABSTRACT: This paper presents the fast implementation of Lock-in Thermography (LIT) before reaching steady-state conditions. The errors caused by the initial temperature drift are compensated by a simple correction formula based on the measurement of the temperature drift image. The effect of this correction has been studied on the in-phase, quadrature, amplitude and the phase signals of lock-in thermography. Simulations and experiments have been performed for this study while considering a specific application of LIT for the investigation of solar cells. This study shows that in our case the error caused by the temperature drift is basically a baseline shift in the in-phase and the quadrature signal, and does not significantly affect the relative shape and spatial resolution of these signals. However, for the amplitude and phase signal this correction is more significant and affects also the relative shape of the signals. Experimental results validate the correction method and its effectiveness in eliminating errors from LIT data measured before thermal equilibrium has been established.

KEY WORDS: lock-in thermography, solar cell, temperature drift correction.
1. Introduction

Lock-in Thermography (LIT) is a non-destructive evaluation technique in which the sample under investigation is heated in a periodic manner and the oscillating surface temperature of the sample is captured by an IR camera. Measured data are averaged over many cycles for detecting the smallest possible thermal contrast due to any abnormality in the sample. LIT can be applied to different types of samples and structures, e.g. to civil structures, pipes, industrial vessels and also electronic devices, with different heating conditions and sources like mechanical, photothermal, or electrical. LIT has been used mostly in non-destructive testing (NDT) of mechanical structures (Maldague, 2001), but has also been used for failure analysis in electronic devices (Wu, 1996; Kuo et al., 1988; Breitenstein et al., 2003).

Figure 1. Typical temperature evolution at the beginning of a lock-in thermography measurement.

The basic principle of LIT investigation is to extract the oscillating part of the surface temperature signal from the background. Fig. 1 shows a typical temperature evolution plot in an LIT measurement for a finite thickness material, which is exposed to periodic square-shaped heating pulses beginning from $t = 0$. The initial temperature evolution is superimposed to a temperature ramp and does not show pure oscillating behaviour. Only after some time, called ‘thermal relaxation time’, the mean temperature of the sample gets stabilised when sample reaches ‘quasi-steady-state condition’ with its surrounding. The thermal relaxation time can vary from a few seconds up to many minutes, depending on the sample thermal properties, dimensions and environmental conditions. For the quantitative evaluation of LIT results, it is generally assumed that the sample is in quasi-steady state condition so that its mean temperature has stabilized. If the time of a LIT measurement is chosen to be large enough compared to the initial heating phase (thermal relaxation time), this assumption is well satisfied. In many cases, however, a long measurement time is not suitable. This holds, for example, in NDT.
investigations of mechanical structures, where the desired signal-to-noise ratio can be obtained already in a measure time being small compared to the thermal relaxation time, as well as for an in-line characterization of electronic devices such as solar cells, which also has to be performed in a possibly short measure time. For such a fast implementation of LIT in a short time compared to the thermal relaxation time, it is essential to understand the effect of the initial heating phase to LIT results and to correct errors due to this unsteady-state, which is the goal of this paper.

Some methods have been proposed in the literature for solving the problem of the initial heating phase. The most obvious solution is to start the LIT data acquisition only after a certain number of excitation pulses have passed, so that quasi-steady state condition has already established (Wu, 1996; e/de/vis, 2003). This method, however, is time-consuming. Another proposal was to start a continuous cooling along with pulsed heating, e.g. by applying a cool air stream (Wu, 1996). The correct dimensioning of this measure is not straightforward and strongly depends on the sample properties and on the excitation intensity. A completely different approach is to apply an analytical or numerical (software-based) evaluation of the measured data within the initial heating phase in order to correct for the inevitable temperature ramp. So it was proposed to fit the measured temperature data to an oscillating function superimposed to a slowly varying time-function and to evaluate only the oscillating part (Mangold, 2000). The limitation in this solution is that this method needs all measured image data and requires a relatively high numerical expense. An alternative method had been proposed (Breitenstein et al., 2003; Gupta et al., 2006), which consists in a simple correction formula, based only on the measurement of the temperature drift image, which is the difference between the IR images before and after a LIT measurement. This formula was derived from a ‘Gedankenexperiment’ (mind game) for the case of homogeneous heating and linear temperature rise, which is described in the annex. It was found that the T-drift induced error does not depend on the slope of the T-ramp but only on the temperature difference before and after the LIT measurement. Therefore, it was predicted that this formula would also remain valid for the realistic case of a non-linear temperature increase and local heat sources. The goal of this contribution is to prove this prediction both by simulations and experimentally.

In this paper, the application of LIT for the detection of local shunts in solar cell is considered as a typical example to study the effect of temperature drift on fast LIT measurements. In this investigation, a solar cell is optically excited and the surface temperature of the cell (captured by an infrared camera) is analyzed by the LIT procedure for finding the location of shunts, which create more heat compared to rest of the region. Simulations and experiments are performed for studying the errors caused by the initial temperature drift and to check the validity of the numerical temperature drift compensation method proposed in (Breitenstein et al., 2003), which can make LIT investigation faster and more accurate.
2. Signal processing

The main aim of the LIT formalism is to evaluate the oscillating part of temperature signal from the noise by averaging over many cycles. This is implemented by using the following mathematical relation (see, e.g., Breitenstein et al., 2003)

\[ S = \frac{1}{t_{\text{int}}} \int_{0}^{t_{\text{int}}} F(t) K(t) \, dt \]  

where \( S \) is a output signal, \( F(t) \) is a detected temperature signal, \( K(t) \) is a correlation function and \( t_{\text{int}} \) is an integration time over which the signal is evaluated. The average value of the correlation functions is zero, therefore the correlation should lead to a suppression of any d.c. part of the signal.

For computer implementation, above equation [1] can be written as

\[ S = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} K_{i,j} F_{i,j} \]  

where \( N \) is total number of lock-in cycles and \( n \) is the number of digitizing events (samples) per lock-in period. It is assumed here that the lock-in periods are synchronized to the digitizing events, and the correlation applies only to complete lock-in periods. The correlation function can be of different type, depending upon the requirement. In most cases, dual-phase harmonic (sine and -cosine) correlation functions are used, since they allow to evaluate only the basic harmonic of the signal, which carries most of the information, while suppressing higher harmonics. Sine correlation measures the signal component in-phase with the basic harmonic of the periodic heat excitation (often square-shaped), which is called the ‘in-phase signal’ or the ‘0° signal’ (\( S_{0°} \)), where -cosine correlation measure the -90° phase-shifted component of the periodic excitation, which is called the ‘quadrature signal’ or the ‘-90° signal’ (\( S_{-90°} \)). The -cosine correlation is often used instead of the +cosine one, since the latter is essentially negative. Mathematically, the lock-in signal correlation can be written as

\[ S_{0°} = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} 2 \sin \left( \frac{2\pi(j-1)}{n} \right) F_{i,j} \]  \[ S_{-90°} = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} -2 \cos \left( \frac{2\pi(j-1)}{n} \right) F_{i,j} \]  

Another advantage of dual-phase harmonic correlation is that it provides the basic constituents of other types of signals, e.g. the amplitude (\( A \)) and the phase (\( \phi \)), which are very popular in thermal NDT of materials and carry important information. Mathematically these signals can be written as
In LIT one has to distinguish between thermally thick and thermally thin samples. If a sample represents a flat sheet, which is thin against the so-called thermal diffusion length $\Lambda$ of the material, then it is called thermally thin, otherwise it is thermally thick. $\Lambda$ is defined as the distance in which a plane thermal wave at a frequency $f$ in the material has decayed to 1/e of its amplitude. It reduces with $1/\sqrt{f}$, and for silicon it is about 3 mm for a lock-in frequency of $f = 3$ Hz. At this frequency, silicon solar cells, having a thickness of about 0.3 mm, are clearly thermally thin. The different LIT signals have different properties. The standard signal is the amplitude signal $A$ [5], which is basically proportional to the local dissipated power density and depends on the thermal properties of the sample. However, it is modulated by the local value of the material’s IR emissivity $\varepsilon$, which is not always known and may be inhomogeneous. The phase signal [6], on the other hand, is a measure of the time delay between the modulated heating power and the T-modulation. It is independent of $\varepsilon$, since it relies on the ratio of the $0^\circ$ and the $-90^\circ$ signal, which are both modulated by $\varepsilon$. Therefore, especially in NDT of materials, the phase signal is often the preferred one used for quantitative evaluation of LIT results. The $0^\circ$ signal ($S^{0\circ}$) [3] shows the best possible spatial resolution, and in thermally thin samples it does not display any homogeneous heat source. This property makes this signal especially appropriate for optically excited shunt investigations on solar cells, since here we are only interested to see the point-like shunts, but not to see the homogeneous heating coming from the homogeneous light irradiation.

For a linear temperature ramp superimposed to periodic heat oscillations, the lock-in correlation [2] still can be calculated analytically (Breitenstein et al., 2003). From the result, the following correction formulas for the $0^\circ$ and the $-90^\circ$ signal have been derived (see annex):

\[
A = \sqrt{(S^{0\circ})^2 + (S^{-90\circ})^2} \tag{5}
\]

\[
\phi = \arctan\left(\frac{-S^{-90\circ}}{S^{0\circ}}\right) \tag{6}
\]

\[
S^{0\circ}_{\text{correct}}(x,y) = S^{0\circ}_{\text{meas}}(x,y) - \frac{\Delta T(x,y)}{N n^2} \sum_{j=1}^{n} j 2 \sin\left(\frac{2\pi(j-1)}{n}\right) \tag{7}
\]

\[
S^{-90\circ}_{\text{correct}}(x,y) = S^{-90\circ}_{\text{meas}}(x,y) + \frac{\Delta T(x,y)}{N n^2} \sum_{j=1}^{n} j 2 \cos\left(\frac{2\pi(j-1)}{n}\right) \tag{8}
\]

The subscript ‘correct’ and ‘meas’ are used for the corrected and the measured (uncorrected) signals, respectively, and $\Delta T(x,y)$ is a total temperature drift in each pixel of the image between start and end of the measurement. It is clear from above equation that, as the number of lock-in cycles ($N$) increases and $\Delta T$ approaches a
limiting value, the correction term becomes smaller and smaller and the corrected signal become closer to the measured signal, which is expected for long-lasting LIT measurements.

3. Simulations

In the simulation, an electro-thermal analogy has been used for estimating the temperature evolution over the solar cell. It is a well known approach for the solution of heat transfer problems to describe the spatially extended thermal system by its equivalent electrical circuit (Cheng et al., 2000). For the simulations, a portion of the solar cell is divided into small square-shaped elements, and based on the thermal properties, each element is modeled by its electrical equivalent circuit, which is finally connected and simulated by the circuit simulator ‘PSpice’ (PSpice 2000; Gupta et al., 2002). A schematic of this modeling approach is given in Fig. 2.

In this modeling, the silicon solar cell has been considered thermally thin, which is a realistic assumption for the range of frequency used in this method. The heat capacity of each element is described by a capacitor, the heat resistances to neighboring elements are described by resistors, the local heat sources are described by current sources, and the local voltages represent the local temperatures. The cell in Fig. 2 is described as thermally insulated from the environment (adiabatic). For describing realistic non-adiabatic conditions, from each element another resistor to ground can be introduced. The heat sources in each element mimic the homogeneous irradiation of light, which is used as an excitation source for non-contacting in-line LIT shunt characterization methods. A local shunt is modeled as an extra heat source at the central element. Simulations have been performed with 30x30 elements, which provide a spatial resolution of 1 mm in a region of 30x30 mm². 0° and -90° results obtained through simulations have been converted into amplitude and phase signals, without and with temperature drift correction, and have been compared to results obtained in the quasi steady-state after the initial heating period is over.

Fig. 3 shows a simulated temperature evolution with a 2.5 Hz homogeneous pulsed heat source distributed over the entire solar cell region, without considering a shunt and any heat resistance to the surrounding. In all simulations 10 frames per lock-in period were used (n = 10), hence the frame rate was assumed to be 25 Hz. In the experiments (see section 4) a frame rate of 160 Hz was used, but due to the limited data handling capacity of the software, only 25 Hz frame rate had to be used for the simulations. This type of staircase increase in temperature was expected, which is close to actual LIT measurement in the initial phase (Fig. 1). This adiabatic condition with homogeneous heating is the worst case condition to check the validity of the temperature drift correction method. In this case, the measured (uncorrected) 0° signal was about -150 % of the corrected (steady-state) signal maximum (which is a pure -90° signal), but the corrected signal was very close to zero (-1.5 % of the steady-state -90° signal), as expected for an extended heat source. This result generally validates the correction method and the entire simulation and computation process. This result was independent of the number of lock-in periods (N) and the number of samples (n).
This correction method has also been used to study the effect of the initial heating phase with a point shunt under non-adibatic conditions. Fig. 4 shows the temperature evolution at the point heat source of 2.5 Hz lock-in frequency located in the centre of the sample. A background heat source of the same frequency for homogenous heating was also considered in this simulation. The assumed thermal relaxation time constant was 2.4 s here. The first 6 lock-in cycles were selected for obtaining the uncorrected and corrected images with different types of signals, which corresponds to a measure time of 2.5 s. Hence, for obtaining these images, the temperature traces simulated for each position by PSpice was entered into [3-6] with and without the correction formula [7-8].
For checking the validity of the correction, also the last 6 cycles data at the end of the simulation (shown in Fig. 4) were used to calculate the steady-state image. In these last cycles, the equilibrium temperature was reached to an accuracy of around 1%.

Fig. 5 shows the corresponding corrected, uncorrected and steady-state in-phase images in the same grey level scale. The difference between the uncorrected and corrected images is remarkable; however there is almost no difference between the corrected and steady-state images, which validates the correction as well as the simulation method. In this figure, contrast and brightness are selected in such a way that the differences become clearly visible. The signal maximum is much sharper than expected from Fig. 5, as shown in Fig. 6 by the profiles of the corrected and uncorrected images across the point shunt.

**Figure 4.** Temperature evolution over a point shunt with surrounding heat resistance.

**Figure 5.** Uncorrected, corrected and steady-state in-phase images for point shunt.
Fig. 6. Profiles of the corrected and uncorrected in-phase images across a point shunt. The steady-state profile is identical to the corrected one in this scale.

Fig. 7 shows the relative error of the corrected and uncorrected profile to the steady-state profile. This error is given in % of the maximum steady-state signal amplitude. It shows a marginal discrepancy between the corrected signal and the steady-state signal, which is much smaller compared to the uncorrected signal. It highlights the importance and significance of correction method. This figure also shows that, at least in our case, the correction is mainly a baseline shift of the uncorrected signal. This is due to the fact that the heating is quite uniform, which causes nearly homogenous temperature drift across the sample. In general, the T-drift correction is always proportional to the local T-drift value, see [7] and [8].

Fig. 7. Difference of the corrected and uncorrected in-phase profiles to the steady-state profile across a point shunt, measured in % of the maximum signal.
Fig. 8 shows the corrected and the uncorrected signal profiles of -90° image. The steady-state profile was very close to the corrected profile therefore it has not been included in this figure. These profiles are much broader than in-phase profiles (Fig. 6), and the homogeneous signal contribution is also visible, as expected. Fig. 9 shows the difference between these profiles and the steady-state profile of -90° image, which gives similar type of results as in the case of in-phase signals (Fig. 7).

**Figure 8.** Profiles of the corrected and uncorrected quadrature images across a point shunt. The steady-state profile is identical to the corrected one in this scale.

**Figure 9.** Difference of the corrected and uncorrected quadrature profiles to the steady-state profile across a point shunt, measured in % of the maximum signal.
The error of the corrected profile with steady-state profile (~1%) is much smaller than of the uncorrected profile with steady-state profile (~8%). Also it shows that the correction is basically a baseline shift to the uncorrected results. One important difference in this case is that we can not guess the value of baseline shift from the uncorrected profiles, but in the case of the in-phase signal this can be done, because we know that the corrected in-phase signal of an extended heat source far from the local source is zero.

Based on the simulated in-phase and quadrature signals, amplitude signals have been obtained by using [5]. Fig. 10 shows corrected and uncorrected amplitude profiles and their difference across the point shunt. It has been seen earlier that the corrected in-phase and the quadrature signals are very close to steady-state signals, therefore the same will also be true for amplitude signals which are derived from these two signals. Since the amplitude signal is a non-linear combination of the in-phase and the quadrature signals, the correction in amplitude signal is not just a baseline shift, which was in in-phase and quadrature signals (Figs. 6-9). Insertion of Fig. 10 shows the difference between the uncorrected and corrected (close to steady-state) amplitude signals, which indicate that the shape of the amplitude signal is considerably affected by the correction. This correction leads to an increase of the amplitude signal in shunt position, which makes shunt detection easier in a short averaging time. The amplitude signal is not only important in functional diagnostics of electronics component but also a widely used representation to display sub-surface defects in materials by thermal non-destructive evaluation.

![Figure 10](image.png)

**Figure 10.** Profiles of the corrected and uncorrected amplitude images across a point shunt (the steady-state profile is identical to the corrected one in this scale) and difference evaluated in % of the maximum signal value.
Similar to amplitude signal, phase is also widely used in NDT, whose importance is already discussed in Section 2. Phase signals have been obtained from the in-phase and the quadrature signals by using [6]. Fig. 11 shows the corrected and uncorrected phase profiles, and their difference. In this case, the relative shape of signal is significantly affected by the correction. Here the uncorrected profile has a sharper peak compared to the corrected profile in shunt position. Only the corrected profile provides the expected phase value of -90° far from the point shunt, where the homogeneous heating is dominating. The correct measurement of the phase is very important for a quantitative analysis of LIT results, as it has already been explained in the previous section.

![Figure 11. Profiles of the corrected and uncorrected phase images across a point shunt (the steady-state profile is identical to the corrected one in this scale) and difference evaluated in % of the maximum signal value.](image)

4. Experiments

Experiments have been performed to support the simulated results on a multicrystalline silicon solar cell having shunts. All experiments have been performed with the commercial TDL 384 M 'Lock-in' thermography system by Thermosensorik (Erlangen), which was extended by an external controller to start the lock-in measurement in a well-defined moment. The basic setup is shown in Fig. 12. In this setup, a solar cell was illuminated by a pulsed LED array whose frequency was synchronized to the frames of the IR camera through the controller, which provides a frequency division. A computer was used to store all captured
frames, hence the in-line lock-in correlation facility of the system was not used here. The front side of solar cell was facing towards the incident radiations, and at the back side temperature evolution over the entire solar cell was recorded by the IR camera. Each frame of the camera was made up of 288x288 pixels, which cover the entire 10x10 cm² sized solar cell. The experiment has been performed with the same lock-in frequency (2.5 Hz) as considered in the simulation. However the frame rate was 160 Hz and the number of samples (frames) per lock-in period was taken 64 instead of 10 (in simulation) for reducing the noise. Similar to the simulation, the first 6 lock-in cycles data have been analyzed to obtain the corrected and uncorrected signals, and the same numbers of cycles are used later to obtain the steady-state signals.

**Figure 12.** Schematic of experimental setup for investigation of shunts in solar cell by illuminated lock-in infrared thermography.

Fig. 13 shows the uncorrected, corrected and steady-state experimental in-phase images of the solar cell in the same grey level. Two major shunts in the cells are clearly visible at the upper right side in all the images. As expected, the corrected and the steady-state images are different from the uncorrected image, and they are similar to each other, which shows the effectiveness of correction and its
importance. Another interesting point in this figure is that the position of bus bars (two vertical lines in uncorrected image) is clearly visible in the uncorrected image, but is hardly visible in corrected and steady-state images. This is due to the fact that this signal is a pure -90° signal, which is completely filtered out only in the corrected or in the steady-state 0° signal. The local minimum of the -90° signal in the position of the bus bars is due to a reduced IR-emissivity in these positions.

For quantitative study, a line profile of the uncorrected, corrected and steady-state images across the smaller shunt (shown in Fig. 13) have been plotted. Figs. 14-17 show these profiles with in-phase, quadrature, amplitude and phase signal, respectively. In all these cases, the corrected profile is close to steady-state signal, which shows that the correction is effective in obtaining the steady-state signal in a shorter time. These figures also show that the effect of correction is more significant in the in-phase, amplitude and phase signals compared to quadrature signal, where the differences are within the experimental error. The reason for this is that the -cosine correlation function of the quadrature (-90°) signal, in contrast to the sine correlation function of the in-phase (0°) signal, is essentially time-symmetric and therefore less affected by a linear T-ramp. In our simulations this effect was less pronounced because of the lower number of frames per lock-in period used there, which also causes a certain time-asymmetry of the -90° correlation.

**Figure 14.** Profiles of the uncorrected, corrected and steady-state in-phase images across a shunt.
Figure 15. Profiles of the uncorrected, corrected and steady-state quadrature images across a shunt.

Figure 16. Profiles of the uncorrected, corrected and steady-state amplitude images across a shunt.
Fig. 18 shows the difference of corrected and uncorrected 0°, -90°, amplitude and phase signals as a percentage of maximum signals. The position of the shunt is marked by an arrow to distinguish it from the peak/dip due to the bus bar of solar cell. As mentioned above, the correction is strongest in the phase and lowest in the -90° image. It should be mentioned that, compared to the steady-state image, the T-drift correction procedure enhances the noise level. This is most clearly visible in the phase profile (Fig. 17). This is due to the noise contribution of the T-drift image $\Delta T$, which is just the difference between two single noisy images. This degradation can be considerably reduced if not single images but the average over a number of images is used to calculate $\Delta T$, as shown by the thick line in Fig. 18 (d) with average of 32 frames.

5. Conclusions

Simulations and experiments have shown that the error of lock-in thermography caused by the drift of the sample temperature can easily be corrected after the measurement by using the temperature drift signal. For avoiding degradation of the signal-to-noise ratio by this procedure, the T-drift signal should be calculated as an average over a number of frames at the beginning and at the end of the measurement. The effect of drift is more significant in the phase, 0°, and amplitude signals compared to the -90° signal. If the sample heats up essentially homogeneously, the drift-induced error is predominantly a baseline shift in the in-phase and
quadrature signals, and therefore does not significantly affect the spatial profile of these signals. However, in the present case, the error caused by drift in the amplitude and phase signals is not only just a baseline shift, but their spatial profiles are significantly affected by the correction. The amplitude signal, which is the most common signal for displaying LIT results, gets enhanced after the correction, which will improve defect (shunt) detection in shorter time.

Altogether it has been shown that the correction method, which uses only the local value of the temperature drift, is an effective and simple way to correct errors due to local heat sources and non-linear temperature drift during the measurements under non steady-state conditions, which is required for a fast implementation of LIT. Results obtained after the correction can be used also for quantitative studies similar to results obtained under steady-state conditions, which usually take considerably longer measure time.

Figure 18. Difference of the corrected and uncorrected profiles, measured in % of the maximum signal value. (a) in-phase (b) quadrature (c) amplitude (d) phase.
Acknowledgments

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6. References


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Appendix

Gedankenexperiment (mind game) for deriving the correction formula [7] and [8]

Consider two LIT experiments $a$ and $b$ on a thermally thin sample under pulsed homogeneous heat introduction. Assume that, initially in experiment $a$, sample is under adiabatic condition (thermally isolated). This leads to rise of sample temperature like a staircase ramp from initial temperature $T_0$ as shown in fig. A1. After a certain time $t_1$ (6 periods) a constant cooling is instantly thought to be switched on. The cooling power is assumed to be equal to the average heating power, so that after time $t_1$ the system instantly reaches to steady-state conditions at an equilibrium temperature $T_{eq}$. In a second experiment $b$, right from the beginning, the same sample is exposed to a cooling power being half the average heating power. In this case, sample reached to a temperature $T_{eq}$ after $t_2 = 12$ periods, and then full cooling power is switched on again leading to steady-state conditions.

![Figure A1](image)

**Figure A1.** Temperature traces of the Gedankenexperiments $a$ and $b$.

The advantage of this Gedankenexperiment is that the lock-in results can be easily calculated. Lock-in signal $S$ of LIT data having $N$ lock-in periods and $n$ samples in each period can be obtained from the following correlation procedure:

$$S = \frac{1}{nN} \sum_{i=1}^{n} \sum_{j=1}^{N} K_{i,j} T_{i,j}$$

[9]
Here $T_{ij}$ is the temperature signal and $K_i$ is a values of the correlation function, which can be written for the in-phase signal $S^0$ and the quadrature signal $S^{90^\circ}$ as:

$$K_j^0 = 2\sin\left(\frac{2\pi(j-1)}{n}\right) \quad K_j^{-90^\circ} = -2\cos\left(\frac{2\pi(j-1)}{n}\right)$$ [10]

Now we split the temperature signals of these two experiments within each lock-in period $i$ into three components: one d.c. component $Td_i^{ab}$ (which is constant within each single period but differs from period to period in the linear slope part of experiments $a$ and $b$), one oscillating component $To_i^{ab}$ (which corresponds to the steady-state signal and is the same in all periods), and one linear slope component $Tl_i^{ab}$, having an average value of zero within each period and a constant slope in the linear slope part of experiments $a$ and $b$, but being zero in the subsequent steady-state parts of the experiments:

$$T_{ij}^{ab} = Td_i^{ab} + To_i^{ab} + Tl_i^{ab}$$ [11]

with

$$Td_i^{ab} = \frac{1}{n} \sum_{j=1}^{n} T_{ij}^{ab}$$ [12]

and

$$Tl_i^{a} = \frac{j - \frac{n}{2} - \frac{1}{2}}{6n} \Delta T \quad [1 \leq i \leq 6], \quad Tl_i^{a} = 0 \quad [i > 6]$$

and

$$Tl_i^{b} = \frac{j - \frac{n}{2} - \frac{1}{2}}{12n} \Delta T \quad [1 \leq i \leq 12], \quad Tl_i^{b} = 0 \quad [i > 12]$$

Here $\Delta T = T_{2i} - T_0$ is the temperature drift value. As mentioned above, the linear slope part of the experiment $a$ has double the amplitude of experiment $b$, but it lasts only for half the number of periods. If [11] with [12] is inserted into [9], all sums containing any d.c. components $Td$ will vanish because of the d.c. rejection properties of the correlation function (the sum over all $K_i$ is zero). The sums containing the linear slope component $Tl$ have to be performed only over the linear slope part of experiments $a$ and $b$. This has to be done with the in-phase and the quadrature correlation according to [10]. The only remaining non-vanishing sums are:

$$S_0^{0^\circ,-90^\circ} = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} To_j^{a} K_j^{0^\circ,-90^\circ} + \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} \frac{j - \frac{n}{2} - \frac{1}{2}}{6n} \Delta T K_j^{0^\circ,-90^\circ}$$ [13]

$$S_{ab}^{0^\circ,-90^\circ} = \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} To_j^{a} K_j^{0^\circ,-90^\circ} + \frac{1}{nN} \sum_{i=1}^{N} \sum_{j=1}^{n} \frac{j - \frac{n}{2} - \frac{1}{2}}{12n} \Delta T K_j^{0^\circ,-90^\circ}$$
The first term are the steady-state lock-in signals $S_{\text{correct}}^{0°-90°}$, which are the same in experiments $a$ and $b$. The second term can easily be summed up over $i$, since the summands do not contain $i$, whereby the factors 6 and 12 are cancelling. So the results for experiments $a$ and $b$ are the same. Regarding the fact that all remaining sums over $j$, where the summand does not contain $j$, are zero (because the sum over all $K_j$ is zero), the lock-in signal for both experiments $a$ and $b$ can be written as:

$$S_{\text{meas}}^{0°-90°} = S_{\text{correct}}^{0°-90°} + \frac{\Delta T}{Nn} \sum_{j=1}^{N} K_j^{0°-90°}$$  \[14\]

Together with [10] this corresponds to [7] and [8].