Chaos supported stochastic resonance in a metal-ferroelectric-semiconductor heterostructure

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An experimental study is presented on a complex nonlinear system showing a particular type of dynamics that can be interpreted as stochastic resonance. The system consists of a metal-ferroelectric-semiconductor structure, which plays the role of a nonlinear element in an electric circuit with linear resistance, inductance, and capacitance connected in series (*RLC* series circuit) driven externally by a high-amplitude harmonic voltage source. The system presents various kinds of nonlinear behavior, of which the simplest, consisting of a period-doubling evolution to chaos, is of interest to this study. The broadband intrinsic chaos emerging after a period-doubling sequence exists for a large range of frequencies of the driving voltage. The appearance of the chaotic dynamics is associated with the promotion of a low-frequency harmonic spectral component. This is interpreted as stochastic resonance with intrinsic chaos replacing noise, the usual variable in regular SR.

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Stochastic resonance is a phenomenon widely investigated during the last 20 years. Initially proposed by Benzi *et al.* [1] in 1981 to explain the problem of the periodically recurrent ice ages, in subsequent studies, it has been repeatedly observed in a large variety of systems. Among these are electronic circuits [2], ring lasers and semiconductor feedback lasers [3], superconducting quantum interference devices (SQUID's) [4], tunnel diodes [5–7], and structures of the neuroscience area [8,9].

In most of these works stochastic resonance is generated by interaction between random (usually Gaussian) noise and a weak periodic signal in a nonlinear system characterized by a bistable potential function: $U(x)=-ax^2+bx^4$, with a and b positive constants.

However, the stochastic process that is necessary for the promotion of the small-amplitude harmonic oscillation in SR is not limited to noise. It has been demonstrated through numerical investigations that external or intrinsic chaotic sources could replace noise. An external chaotic source applied to a periodically driven Duffing oscillator in a regime where it produces period-doubling dynamics yields switching between attractors corresponding to two phase-shifted responses of the Duffing oscillator [10,11]. On the other hand, by use of the intrinsic chaotic dynamics of a nonlinear map in the vicinity of a band-merging crisis, it is possible to generate an activated hopping process that is then synchronized by a small periodic signal [12]. An experimental observation of deterministic stochastic resonance has been obtained in a bistable semiconductor system based on p-Ge(Au) systems [13].

Another important extension of this phenomenon is stochastic resonance in coupled nonlinear systems. Two coupled bistable systems showed clear resonance in periodic response with coupling constants or noise strength tuning [14].

A comprehensive review of SR can be found in [15] and a semipopular level presentation in [16].

The system that is investigated in this work is a metal-ferroelectric-semiconductor (MFS) structure, which presents a higher level of complexity compared with metal-ferroelectric-metal structures whose chaotic behavior as nonlinear element of an *RLC* series circuit has been extensively described previously [17,18]. In our case, the combination between the strongly nonlinear behavior of the ferroelectric film and the highly asymmetric electrical properties of metal-oxide-semiconductor (MOS)–like structures considerably enhances the expected complexity of the dynamics. Some features of the qualitatively new picture obtained are addressed in the present work.

The experimental setup shown in Fig. 1 consists of an *RLC* series circuit with the MFS structure acting as nonlinear capacitance in series with linear components, a capacitance C_0, a resistance R, and an inductance L. The MFS structure was fabricated by sol-gel deposition of a $Bi_4Ti_3O_{12}$ (BIT) film on a p-type Si(100) substrate with a resistivity of $1-10\ \Omega$ cm. The structures were annealed for 30 min at 650 °C in air. The thickness (286 ± 15 nm) of the BIT film has been determined by scanning electron microscopy (SEM). X-ray-diffraction (XRD) analysis showed a polycrystalline structure for the ferroelectric layer. The electrical contacts have been realized by vacuum evaporation of Al. The top contact (on the BIT material) was profiled using a mask while, on the Si substrate, Al was evaporated on the entire surface. Further information on the sample fabrication can be found in [19].

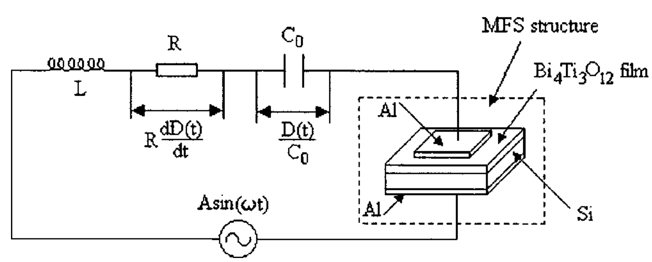

FIG. 1. Diagram of the *RLC* series circuit driven by a sinusoidal voltage source. The nonlinear element (MFS structure) is shown in the box drawn with the dashed line.

The series *RLC* circuit was externally driven by a harmonic voltage generator type HP8116A. Time series acquisitions of voltage drops on the linear capacitor and the resistor were performed by an oscilloscope TDS 540A with sampling periods between 0.2 and 2 μs and 8-digit precision. The voltage drop on the resistance is proportional to the derivative with respect to time of the dielectric displacement D, while the voltage drop on the linear capacitor is proportional to D. The entire circuitry was shielded in a metallic box against external electromagnetic perturbations.

The length of the time series was 15 000 points. Their spectral analysis was performed by fast Fourier transform (FFT). In order to obtain clear resonances and good estimation of their spectral power a Hanning window was used.

In previous works dedicated to the dynamics of this system [20,21], we presented sequences of period-doubling bifurcations leading to chaotic behavior at constant frequency of the harmonic forcing, the amplitude thereof playing the role of control parameter, as well as similar sequences with frequency as control parameter at constant amplitude.

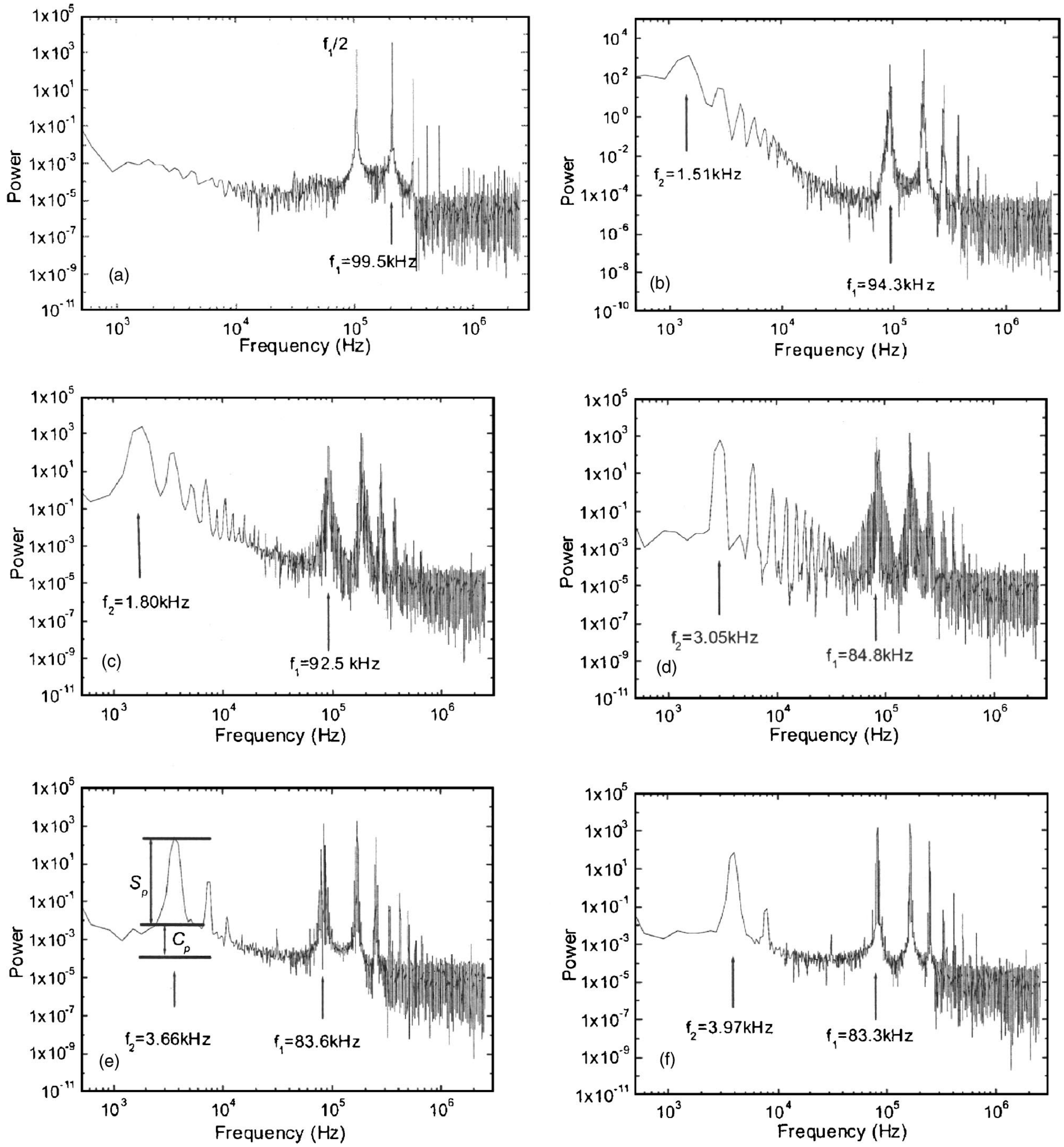

FIG. 2. Power spectra of the voltage drop on the linear capacitance at constant amplitude (12 V) for different frequencies of the driving signal marked on each graph by an arrow and denoted f_1. The frequency f_2 of the promoted component is also specified.

In this study we present aspects of the dynamics of the system in the chaotic regime following a period-doubling bifurcation sequence. For the amplitude of the forcing of 12 V used in this study, the chaotic behavior is reached at a frequency of about 96 kHz and consists of a high-power continuous spectrum especially in the range below the forcing frequency. We observe the following interesting aspects of the chaotic behavior: first, chaotic dynamics persists for an extensive range of frequencies of the driving voltage below 96 kHz, and second, the level of the chaotic component is dependent on this frequency. However, the main observation refers to the appearance of a low-frequency component practically simultaneously with the setting in of the chaotic behavior. It can be considered that intrinsic chaos is promoting this component.

All the dynamics described is clear from the graphs in Fig. 2. Figure 2(a) presents the spectrum for an arbitrary situation in the range of regular dynamics (particularly the first period doubling bifurcation) and is only given for reference. The spectrum for a frequency slightly lower than 96 kHz is shown in Fig. 2(b). Here, above the continuous spectrum, an additional low-frequency harmonic component can be observed. As this component is not present in the spectra corresponding to regular dynamics [Fig. 2(a)], we consider it to be promoted by the presence of the chaotic dynamics.

The spectra in Figs. 2(b)–2(f) show that the level of the chaotic component, as measured from the base level of the regular dynamics spectrum [Fig. 2(a)], is changing with the frequency of the driving voltage. In all these spectra we observe the presence of a low-frequency component (below 6 kHz) denoted f_2, much smaller than the driving frequency denoted f_1. For different levels of the chaotic component, the amplitude of the additional low-frequency spectral component f_2 is different.

We consider these experimental findings as evidence for chaos-supported stochastic resonance, where chaos plays the same role as noise in conventional nonlinear systems showing SR.

This behavior suggested the possibility of studying the variation of the signal to chaos ratio as function of the chaos level.

Spectra of the type presented in Fig. 2 were used to generate the diagram in Fig. 3, which shows the signal/chaos ratio (S/C), as a function of the chaos level.

The signal/chaos ratio is defined by

$$\frac{S}{C} = 10 \ln\left(\frac{S_P}{C_P}\right), \tag{1}$$

where S_P represents the power of the promoted frequency (f_2) and C_P is the power of the chaotic component at the same frequency. Here, we estimate S/C using the approximate method presented on Fig. 2(e). S_P is measured from the chaos level at the frequency f_2 while C_P is measured from the base level of the spectra for regular behavior [Fig. 2(a)]. This estimation of the magnitudes of S_P and C_P is good if the peak at frequency f_2 is very narrow. In the case of our graphs this algorithm represents a more modest approximation; however, we consider it satisfactory for the qualitative analysis presented in this paper.

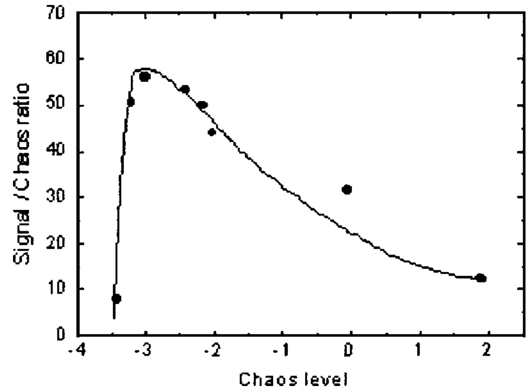

FIG. 3. Signal-to-chaos ratio as function of the chaos level, suggesting stochastic resonance. Definition of the scale units is given in the main text. The solid line is drawn simply for guiding of the eye.

The abscissa in Fig. 3 represents the chaos level defined as the logarithm to base 10 of C_P. By using a logarithmic scale, the visibility of the maximum of the S/C versus the chaos level curve is considerably enhanced. The dots represent values calculated from spectra obtained for various frequencies of the harmonic driving of the type shown in Fig. 2. The solid line is drawn simply for guiding the eye.

The shape of the curve in Fig. 3 bears a striking similarity to equivalent curves representing the signal-to-noise ratio as function of the noise power obtained for conventional systems showing SR—e.g., double-well potential systems [15] (and references therein). This observation can be considered as further proof for the validity of our interpretation of the experimental results as chaos-supported stochastic resonance.

In conclusion, this work presents a large-signal analysis of the dynamics of a system not investigated previously from this point of view. A metal-ferroelectric-semiconductor structure plays the role of a nonlinear element in an *RLC* series circuit driven externally by a large-amplitude harmonic voltage source. The system presents various kinds of nonlinear dynamics but in this work we only refer to the chaotic behavior generated by a period-doubling sequence.

Simultaneously with the appearance of the chaotic dynamics, a low-frequency spectral component is promoted. The dependence of the amplitude of this oscillation on the chaos level suggests the interpretation of the phenomenon as stochastic resonance where the chaotic component replaces noise, the usual variable in conventional SR. This hypothesis is further supported by the similarity of the dependence of the signal-to-chaos ratio on the chaos level with the curve of signal-to-noise ratio versus noise power in conventional SR.

It is clear that, for different levels of the chaotic component, the frequency of the promoted spectral component f_2 is slightly different. We did not yet manage to find any clear relationship between f_2 and the driving frequency f_1. The question of the origin of this oscillation, as well as the influence of relevant parameters of the system on the type of dynamics presented in this paper, constitutes issues for further investigation.

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