Pure Spin Currents and the Associated Electrical Voltage

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We present a generalized Landauer-Büttiker transport theory for multiterminal spin transport in the presence of spin-orbit interaction. Using this theory we point out that there exists equilibrium spin currents and nonequilibrium pure spin currents, without any magnetic element in the system. Quantitative results are presented for a Y-shaped conductor. It is shown that pure spin currents cause a voltage drop and, hence, can be measured.

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Producing and measuring *spin currents* is a major goal of spintronics. The standard way is to inject spin currents from a ferromagnet into a semiconductor in a two terminal geometry [1]. However, this has a drawback, due to conductivity mismatch: The polarization of the injected current is rather small and it always has an accompanying charge current [2]. Also, for any spintronics operation, the spin-orbit interaction plays an important role, e.g., in the Datta-Das spin transistor [3].

In light of these developments, it would be interesting and highly desirable if one could produce spin currents intrinsically. One such possibility is provided by an intrinsic spin-orbit interaction. The presence of an impurity atom or defects gives rise to spin-orbit interaction of the form [4,5]

$$H_{so} = \lambda[\nabla U(r) \times \mathbf{k}] \cdot \boldsymbol{\sigma}, \tag{1}$$

where $\boldsymbol{\sigma}$ is a vector of Pauli matrices, $\mathbf{U}(r)$ is the potential due to defects or impurity atoms, $\mathbf{k}$ is the momentum wave vector of electrons, and λ is spin-orbit interaction strength. For the strictly two-dimensional case for which the potential $\mathbf{U}(r)$ depends on x and y coordinates, only the Hamiltonian commutes with σ_z; hence, the z component of the spin is a good quantum number. As is well known, this kind of spin-orbit interaction has a polarizing effect on particle scattering [6]; i.e, when an unpolarized beam is scattered it gets polarized perpendicular to the plane of scattering. Further scattering of this polarized beam causes asymmetry in scattering processes; i.e., electrons with one particular spin direction, e.g., spin-up electrons, have a larger probability to be scattered to the right compared to spin-down electrons [5,6]. This property of spin-orbit scattering gives rise to novel effects such as the spin Hall effect [4].

In this Letter, we show that the above discussed property of spin-orbit scattering can be used to generate and measure spin currents. Consider a three terminal, two-dimensional Y-shaped conductor shown in Fig. 1. The plane of the conductor is xy. Since the conductor is two dimensional, which fixes the scattering plane, the scattered electrons will be polarized along the z axis (perpendicular to the scattering plane). However, the polarization for the two branches of the Y junction will be opposite [4]. Hence, a three terminal structure would create spin currents from an unpolarized current in the presence of spin-orbit interaction [5,7]. Moreover, a three terminal device provides an important possibility of generating nonequilibrium pure spin currents without an accompanying charge current. This happens when one of the terminals acts as a voltage probe. Say the terminal 3 is a voltage probe as shown in Fig. 1, i.e., the voltage V_3 at the third probe is adjusted such that the total charge current flowing in terminal 3 is zero; i.e., $I_3^q = 0$ [8,9]. Physically, it implies that the charge current flowing in (which is polarized as argued above) is the same as the charge current flowing out. However, the polarization of the charge current flowing out need not to be same as the polarization of the charge current flowing in (see Fig. 1). Hence, there will be a net spin current flowing without the

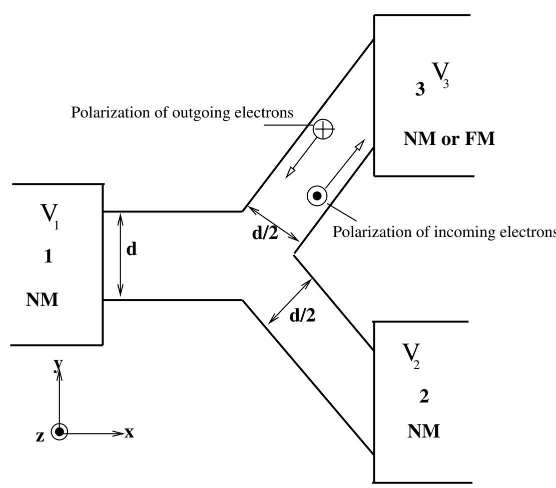

FIG. 1. Y-shaped three terminal junction with applied voltages V_1, V_2, and V_3 as depicted. The third terminal (labeled 3) is a voltage probe (nonmagnetic or ferromagnetic) which draws no charge current. However, the polarization of incoming and outgoing electrons are opposite to each other, causing a *pure spin current.*

accompanying charge current. This is a pure nonequilibrium spin current. We show that this pure spin current can be detected when the voltage probe is nonmagnetic and can be measured, if the voltage probe is magnetic.

The possibility of injecting pure spin currents was first discussed in Ref. [10] for a three terminal device where two of the terminals were ferromagnetic. Also, direct optical injection of pure spin currents in GaAs/AlGaAs quantum wells was demonstrated in Ref. [11]. We would stress that in our case spin current is not injected, rather generated intrinsically due to the spin-orbit interaction without any magnetic element in the system, which is not the case in Ref. [10]. Thus, we avoid the problem of spin injection altogether. Further, since the effect discussed relies on the general scattering properties due to the spin-orbit interaction, it will be observable with any kind of spin-orbit interaction, e.g., the Rashba spin-orbit interaction [12].

We first briefly outline the spin transport theory for multiterminal devices. Let us choose the spin quantization axis to be along $\hat{\mathbf{u}}$, pointing along (θ, ϕ), where θ and ϕ are usual spherical angles (in other words, we choose the spin basis to be eigenstates of operator $\boldsymbol{\sigma} \cdot \hat{\mathbf{u}}$). This is essential since a charge current flowing along a spatial direction can be polarized along a direction which need not coincide with the direction of flow of charge current. Also, in the presence of spin-orbit interaction, the rotational invariance in spin space is lost [13]; hence, any theory for spin transport should take this fact into account. With this definition, we can generalize the Landauer-Büttiker theory for spin transport. Let V_m be a potential at a terminal m measured from the minima of the lowest band, where m can take values 1, 2, and 3 corresponding to the three terminals of Fig. 1. Let N_m^σ and $N_m^{-\sigma}$ be the number of channels for two spin components in lead m. $T_{nm}^{\alpha,\sigma}$ is the spin-resolved transmission probability of electrons incident in lead m in spin channel σ to be transmitted into lead n in spin channel α and $R_{mm}^{\alpha,\sigma}$ is the corresponding reflection probabilities. Charge currents in spin channel σ that impinge on the sample from terminal m are $I_{mm}^\sigma = (e^2/h)[N_m^\sigma - (R_{mm}^{\sigma\sigma} + R_{mm}^{-\sigma\sigma})V_m]$. This current leaves the sample through other leads (since this is also a charge current which should go somewhere); i.e., $I_{mm}^\sigma = \sum_{n\neq m,\alpha} I_{nm}^{\alpha\sigma} = (e^2/h)\sum_{n,\alpha} T_{nm}^{\alpha\sigma} V_m$. Lead n causes a current $-(e^2/h)\sum_\alpha T_{mn}^{\sigma\alpha} V_n$ in lead m in spin channel σ. Hence, the net spin current σ flowing into terminal m is

$$I_m^\sigma = \frac{e^2}{h} \sum_{n\neq m,\alpha} (T_{n\,m}^{\alpha\,\sigma} V_m - T_{m\,n}^{\sigma\,\alpha} V_n). \tag{2}$$

In writing the above equations, we have made an assumption that the spin-resolved transmission coefficient is energy independent. The generalization of the above equation, when the spin-resolved transmission coefficient is energy dependent, is straightforward.

Since SO interaction preserves time reversal symmetry, this leads to the following constraint on the spin-resolved transmission coefficient:

$$T_{n\,m}^{\alpha\,\sigma} = T_{m\,n}^{-\sigma\,-\alpha}. \tag{3}$$

Using Eq. (2), we can immediately write down the net charge and spin current flowing through terminal m:

$$I_m^q = I_m^\sigma + I_m^{-\sigma} \equiv \frac{e^2}{h} \sum_{n\neq m,\sigma,\alpha} (T_{n\,m}^{\alpha\,\sigma} V_m - T_{m\,n}^{\sigma\,\alpha} V_n), \tag{4}$$

$$\begin{aligned} I_m^s &= I_m^\sigma - I_m^{-\sigma} \\ &\equiv \frac{e^2}{h} \sum_{n\neq m,\alpha} \{(T_{n\,m}^{\alpha\,\sigma} - T_{n\,m}^{\alpha\,-\sigma})V_m + (T_{mn}^{-\sigma\,\alpha} - T_{mn}^{\sigma\,\alpha})V_n\}, \end{aligned} \tag{5}$$

where I_m^q is charge current and I_m^s is spin current. We stress that Eq. (5) correctly determines spin current generated by the presence of spin-orbit interaction. In the absence of spin-orbit interaction and any magnetic element in the device, the spin-resolved transmission coefficient obeys a further rotational symmetry in spin space, i.e., $T_{n\,m}^{\alpha,\sigma} = T_{n\,m}^{-\alpha,-\sigma}$ and $T_{n\,m}^{-\sigma,\sigma} = 0$ (spin flip transmission probabilities are zero in the absence of SO interaction), which implies that spin currents are identically zero for all terminals (nonmagnetic), i.e., $I_m^s = 0$.

Equilibrium spin current.—To discuss equilibrium spin currents, let us consider the case when all the potentials are equal, i.e., $V_m = V_0 \forall\, m$. In this situation, the charge current flowing in any terminal should be zero ($I_m^q = 0$) which leads to the following sum rule [from Eq. (4)]:

$$\sum_n T_{nm} = \sum_n T_{mn}, \tag{6}$$

where $T_{nm} = \sum_{\alpha,\sigma} T_{nm}^{\alpha,\sigma}$ is total transmission probability (summed over all spin channels) from terminal m to n. This sum rule is robust and should be satisfied irrespective of the detailed physics [14]. This is a well-known gauge invariance condition. Charge conservation implies $\sum_m I_m^q = 0$, which follows from the symmetry of the spin-resolved transmission coefficient, Eq. (3), and the gauge invariance condition, Eq. (6). In equilibrium there are no charge currents flowing. However, this is not the case for the spin currents. This point can be appreciated if we look closely at Eq. (5) for spin current. Since in general the transmission coefficient $T_{n\,m}^{\alpha\,\sigma} \neq T_{n\,m}^{\alpha\,-\sigma}$, which occurs in Eq. (5), even when all the potentials are equal, the spin current given by Eq. (5) is nonzero. This is equilibrium spin current. Notice that this is consistent with time reversal invariance [Eq. (3)] and the gauge invariance condition given by Eq. (6). We would like to point out that this equilibrium spin current would exist even in a two terminal setup without any magnetic element in the system. Slonczewski showed in Ref. [15] that equilibrium spin currents cause nonlocal exchange coupling in a two terminal geometry with two magnetic contacts. The important difference in our case is that we do not need ferromagnetic contact to have equilibrium spin currents, which was the case in Ref. [15]. The equilibrium spin currents are carried by all the occupied states at a given temperature. Strictly speaking, for the equilibrium spin currents, one should take into account the energy dependence of spin-resolved transmission coefficient. A detailed study of the equilibrium spin currents will be presented in a separate article [16]. In this study, we concentrate more on the nonequilibrium pure spin currents and the related electrical effects.

Nonequilibrium spin currents.—To study nonequilibrium spin currents, let us consider the case where the voltages at terminals 1 and 2 are, respectively, $V_1 = 0$ and V_2, and the third terminal is a voltage probe, i.e., $I_3^q = 0$. With this condition, one can determine the voltage, V_3, at the third terminal using the set of Eq. (4) and given by [9]

$$\frac{V_3}{V_2} = \frac{T_{32}}{T_{13} + T_{23}}. \tag{7}$$

The spin current flowing through terminal 3 is

$$I_3^s = \frac{e^2}{h}\sum_\alpha \{(T_{13}^{\alpha\sigma} - T_{13}^{\alpha-\sigma} + T_{23}^{\alpha\sigma} - T_{23}^{\alpha-\sigma})V_3 + (T_{32}^{\alpha-\sigma} - T_{32}^{\alpha\sigma})V_2\}. \tag{8}$$

From Eq. (8), it is clear that I_3^s is nonzero, while I_3^q is zero by definition. Hence, in terminal 3 there are net spin currents flowing in the absence of any net charge current. This is pure spin current and is intrinsically generated by the spin-orbit interaction in the absence of any magnetization as discussed in the introduction.

To obtain quantitative results, we perform a numerical simulation on a Y-shaped conductor shown in Fig. 1. We model the conductor on a square tight binding lattice with lattice spacing a, and we use the corresponding tight binding model including the spin-orbit interaction given by Eq. (1) [5]. For the calculation of the spin-resolved transmission coefficient, we use the recursive green function method. Details of this can be found in Refs. [5,13]. The numerical result presented takes the quantum effect and multiple scattering into account. For the model of disorder, we take Anderson model, where on-site energies are distributed randomly within $[-U/2, U/2]$, where U is the width of distribution.

In Fig. 2, we show the spin currents I_s^3 flowing through the nonmagnetic terminal 3 (right panel) and the corresponding voltage V_3 when all three terminals are nonmagnetic. In Fig. 2, $\theta = 0$ corresponds to the z axis and $\theta = 90$ corresponds to the y axis, we have kept fixed $\phi = 90$. We see that the maximum amount of spin currents flows along the z axis. This is understandable since, for the strictly two-dimensional case, the spin-orbit coupling given by Eq. (1) conserves the z component of spin.

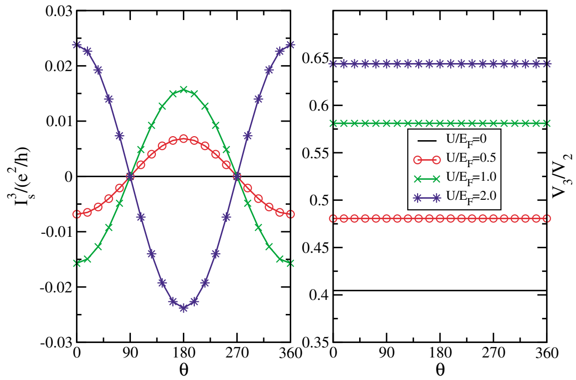

FIG. 2 (color online). Pure spin current flowing through a nonmagnetic voltage probe and the corresponding voltage (right panel) versus quantization axis. Disorder potential strength is shown in the inset. Calculations were performed on a device width $d = 20a$ (see Fig. 1), $k_F a = 1$, dimensionless spin-orbit parameter $\lambda/a^2 = 0.05$.

Hence, the asymmetric scattering produced by spin-orbit interaction causes a pure spin current along the z axis, as discussed in the introduction. For the ballistic case (curve for $U/E_F = 0$), spin currents are zero since there is no spin-orbit interaction in this case, as can be seen from Eq. (1) by putting the potential $U(r) = 0$. Also, for strong disorder, spin current changes sign (curve for $U/E_F = 2$) due to multiple scattering. From the right panel of Fig. 2, we see that the voltage V_3 measured is different although there is no charge current flowing and the magnitude of V_3 is directly proportional to the z component of spin current. As seen, with the increase of disorder strength, the magnitude of the spin currents increases and, accordingly, the potential V_3 also increases. However, potential V_3 is independent of the quantization axis since the voltage probe is nonmagnetic. Hence, with a nonmagnetic voltage probe, one can detect the spin current, but cannot measure it. To measure the spin currents, one would need a ferromagnetic voltage probe. An intuitive understanding of this can be gained as follows. From Fig. 2 (right panel), we notice that the spin currents depend on the quantization axis. Thus, if the probe is ferromagnetic, electrons which are polarized parallel to the ferromagnet would be transmitted more easily than the electrons polarized antiparallel to the ferromagnet. Since the voltage at the probe is determined by the ratio of the transmission coefficient [Eq. (7)], the probe voltage should show variation in phase with the spin currents.

This is confirmed in Fig. 3, where we have plotted spin current (left panel) and voltage (right panel) for the case when the third terminal is a ferromagnetic. The quantization axis is given by the direction of magnetization. We see that, as the spin current changes, the corresponding voltage measured also changes in phase. Hence, by having a ferromagnetic voltage probe one can measure the pure spin current. We would like to mention that in our numerical simulation the voltage probe is an invasive one;

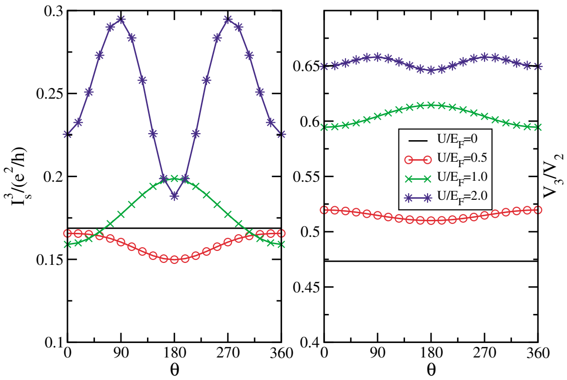

FIG. 3 (color online). Pure spin current flowing through the ferromagnetic voltage probe versus quantization axis (left panel); right panel shows the corresponding voltage. Disorder potential strength is shown in the inset. Ferromagnet is modeled as exchange split with exchange splitting (Δ) given as $\Delta/E_F = 0.5$. Other parameters are the same as for Fig. 2.

i.e., it is strongly coupled to the system; hence, one sees the quantitative and qualitative difference between the results of Figs. 2 and 3. This is so because the ferromagnetic probe is strongly coupled (invasive probe); it essentially injects a polarized current and disturbs the system. This can be overcome by using a weakly coupled (noninvasive) voltage probe which would allow one to measure spin currents just due to SO coupling.

The parameters used in the numerical calculation were estimated in the following way. The weak localization measurements provide spin flip scattering time τ_{so}, which falls in the range $\tau_{so}/\tau_{el} \approx 5$–10 [17], where τ_{el} is elastic scattering time. Thus, we estimate λ/a^2 (the spin-orbit parameter in our model) to lie between 0.03 and 0.07. In our simulation we have taken $\lambda/a^2 = 0.05$. We have varied U (on-site potential) in our simulation, since in a real experimental situation it is the concentration of impurities which is controlled. This not only affects SO coupling, it also increases elastic scattering. Hence, it is appropriate to vary U in the simulation. Finally, we can estimate from Fig. 3 that spin currents change upon the magnetization rotation by approximately 5%; a corresponding change in V_3/V_2 is about 3% which is of the same order of magnitude. We stress that thermal disturbances will not reduce spin currents since τ_{so} is essentially temperature independent, which is responsible for spin currents. However, since the effect relies on mesoscopic phase coherent transport, it will be controlled by phase coherence length L_ϕ, which can reach the order of μm at $T = 1$ K. Since SO interaction was shown to reduce conductance fluctuation [18], this will only help in measuring the proposed effect.

In conclusion, we have generalized the Landauer-Büttiker theory for spin transport in the presence of SO interaction. We have proposed a way to generate and measure spin currents in an electrical transport measurement, since charge transport for the Y-shaped mesoscopic junction has been studied in the past experimentally as well theoretically. In view of this, we hope the study presented here for the spin transport will open up new opportunities in the field of spintronics.

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